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**PyBLP**  
*Release 0.10.1*

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## **Part I**

# **User Documentation**



## INTRODUCTION

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**Note:** This package is in beta. The API may change in future versions. Please use the [GitHub issue tracker](#) to report bugs or to request features.

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PyBLP is a Python 3 implementation of routines for estimating the demand for differentiated products with BLP-type random coefficients logit models. This package was created by [Jeff Gortmaker](#) in collaboration with [Chris Conlon](#).

Development of the package has been guided by the work of many researchers and practitioners. For a full list of references, including the original work of [Berry, Levinsohn, and Pakes \(1995\)](#), refer to the [references](#) section of the documentation.

### 1.1 Citation

If you use PyBLP in your research, we ask that you also cite [Conlon and Gortmaker \(2020\)](#), which describes the advances implemented in the package.

### 1.2 Installation

The PyBLP package has been tested on [Python](#) versions 3.6, 3.7, and 3.8. The [SciPy instructions](#) for installing related packages is a good guide for how to install a scientific Python environment. A good choice is the [Anaconda Distribution](#), since it comes packaged with the following PyBLP dependencies: [NumPy](#), [SciPy](#), [SymPy](#), and [Patsy](#). For absorption of high dimension fixed effects, PyBLP also depends on its companion package [PyHDFE](#), which will be installed when PyBLP is installed.

However, PyBLP may not work with old versions of its dependencies. You can update PyBLP's Anaconda dependencies with:

```
conda update numpy scipy sympy patsy
```

You can update PyHDFE with:

```
pip install --upgrade pyhdfc
```

You can install the current release of PyBLP with [pip](#):

```
pip install pyblp
```

You can upgrade to a newer release with the `--upgrade` flag:

```
pip install --upgrade pyblp
```

If you lack permissions, you can install PyBLP in your user directory with the `--user` flag:

```
pip install --user pyblp
```

Alternatively, you can download a wheel or source archive from [PyPI](#). You can find the latest development code on [GitHub](#) and the latest development documentation [here](#).

## 1.3 Other Languages

Once installed, PyBLP can be incorporated into projects written in many other languages with the help of various tools that enable interoperability with Python.

For example, the [reticulate](#) package makes interacting with PyBLP in R straightforward:

```
library(reticulate)
pyblp <- import("pyblp")
pyblp$options$flush_output <- TRUE
```

Similarly, [PyCall](#) can be used to incorporate PyBLP into a Julia workflow:

```
using PyCall
pyblp = pyimport("pyblp")
```

The `py` command serves a similar purpose in MATLAB:

```
py.pyblp
```

## 1.4 Features

- R-style formula interface
- Bertrand-Nash supply-side moments
- Multiple equation GMM
- Demographic interactions
- Micro moments that match demographic expectations and covariances
- Second choice micro moments that match probabilities and covariances
- Fixed effect absorption
- Nonlinear functions of product characteristics
- Concentrating out linear parameters
- Normal and lognormal random coefficients
- Parameter bounds and constraints
- Random coefficients nested logit (RCNL)
- Approximation to the pure characteristics model
- Varying nesting parameters across groups



- Logit and nested logit benchmarks
- Classic BLP instruments
- Differentiation instruments
- Optimal instruments
- Tests of overidentifying and model restrictions
- Parametric bootstrapping post-estimation outputs
- Elasticities and diversion ratios
- Marginal costs and markups
- Profits and consumer surplus
- Merger simulation
- Custom counterfactual simulation
- Synthetic data construction
- SciPy or Artleys Knitro optimization
- Fixed point acceleration
- Monte Carlo, quasi-random sequences, quadrature, and sparse grids
- Importance sampling
- Custom optimization and iteration routines
- Robust and clustered errors
- Linear or log-linear marginal costs
- Partial ownership matrices
- Analytic gradients
- Finite difference Hessians
- Market-by-market parallelization
- Extended floating point precision
- Robust error handling

## 1.5 Features Slated for Future Versions

- Fast, “Robust,” and Approximately Correct (FRAC) estimation
- Analytic Hessians
- Mathematical Program with Equilibrium Constraints (MPEC)
- Generalized Empirical Likelihood (GEL)
- Discrete types
- Newton methods for computing equilibrium prices

## 1.6 Bugs and Requests

Please use the [GitHub issue tracker](#) to submit bugs or to request features.

## NOTATION

The notation in PyBLP is a customized amalgamation of the notation employed by *Berry, Levinsohn, and Pakes (1995)*, *Nevo (2000)*, *Morrow and Skerlos (2011)*, *Grigolon and Verboven (2014)*, and others.

### 2.1 Indices

Index	Description
$j$	Products
$t$	Markets
$i$	Agents/individuals
$f$	Firms
$h$	Nests
$c$	Clusters
$m$	Micro moments

## 2.2 Dimensions/Sets

Dimension/Set	Description
$T$	Markets
$N$	Products across all markets
$F$	Firms across all markets
$I$	Agents across all markets
$J_t$	Products in market $t$
$F_t$	Firms in market $t$
$J_{ft}$	Products produced by firm $f$ in market $t$
$I_t$	Agents in market $t$
$K_1$	Demand-side linear product characteristics
$K_1^{\text{ex}}$	Exogenous demand-side linear product characteristics
$K_1^{\text{en}}$	Endogenous demand-side linear product characteristics
$K_2$	Demand-side nonlinear product characteristics
$K_3$	Supply-side product characteristics
$K_3^{\text{ex}}$	Exogenous supply-side product characteristics
$K_3^{\text{en}}$	Endogenous supply-side product characteristics
$D$	Demographic variables
$M_D$	Demand-side instruments
$M_S$	Supply-side instruments
$M_M$	Micro moments
$T_m$	Markets over which micro moment $m$ is averaged
$T_{mn}$	Markets over which micro moments $m$ and $n$ are both averaged
$M$	All moments
$E_D$	Absorbed dimensions of demand-side fixed effects
$E_S$	Absorbed dimensions of supply-side fixed effects
$H$	Nesting groups
$J_{ht}$	Products in nesting group $h$ and market $t$
$C$	Clusters
$J_{ct}$	Products in cluster $c$ and market $t$

## 2.3 Matrices, Vectors, and Scalars

Symbol	Dimensions	Description
$X_1$	$N \times K_1$	Demand-side linear product characteristics
$X_1^{\text{ex}}$	$N \times K_1^{\text{ex}}$	Exogenous demand-side linear product characteristics
$X_1^{\text{en}}$	$N \times K_1^{\text{en}}$	Endogenous demand-side linear product characteristics
$X_2$	$N \times K_2$	Demand-side Nonlinear product characteristics
$X_3$	$N \times K_3$	Supply-side product characteristics
$X_3^{\text{ex}}$	$N \times K_3^{\text{ex}}$	Exogenous supply-side product characteristics
$X_3^{\text{en}}$	$N \times K_3^{\text{en}}$	Endogenous supply-side product characteristics
$\xi$	$N \times 1$	Unobserved demand-side product characteristics
$\omega$	$N \times 1$	Unobserved supply-side product characteristics
$p$	$N \times 1$	Prices
$s(s_{jt})$	$N \times 1$	Market shares
$s(s_{ht})$	$H \times 1$	Group shares in a market $t$
$s(s_{ijt})$	$N \times I_t$	Choice probabilities in a market $t$

Continued on next page

Table 1 – continued from previous page

Symbol	Dimensions	Description
$c$	$N \times 1$	Marginal costs
$\tilde{c}$	$N \times 1$	Linear or log-linear marginal costs, $c$ or $\log c$
$\eta$	$N \times 1$	Markup term from the BLP-markup equation
$\zeta$	$N \times 1$	Markup term from the $\zeta$ -markup equation
$\mathcal{H}$	$J_t \times J_t$	Ownership or product holdings matrix in market $t$
$\kappa$	$F_t \times F_t$	Cooperation matrix in market $t$
$\Delta$	$J_t \times J_t$	Intra-firm matrix of (negative) demand derivatives in market $t$
$\Lambda$	$J_t \times J_t$	Diagonal matrix used to decompose $\eta$ and $\zeta$ in market $t$
$\Gamma$	$J_t \times J_t$	Another matrix used to decompose $\eta$ and $\zeta$ in market $t$
$d$	$I_t \times D$	Observed agent characteristics called demographics in market $t$
$\nu$	$I_t \times K_2$	Unobserved agent characteristics called integration nodes in market $t$
$w$	$I_t \times 1$	Integration weights in market $t$
$\delta$	$N \times 1$	Mean utility
$\mu$	$J_t \times I_t$	Agent-specific portion of utility in market $t$
$\epsilon$	$N \times 1$	Type I Extreme Value idiosyncratic preferences
$\bar{\epsilon}(\bar{\epsilon}_{ijt})$	$N \times 1$	Type I Extreme Value term used to decompose $\epsilon$
$\bar{\epsilon}(\bar{\epsilon}_{iht})$	$N \times 1$	Group-specific term used to decompose $\epsilon$
$U$	$J_t \times I_t$	Indirect utilities
$V(V_{ijt})$	$J_t \times I_t$	Indirect utilities minus $\epsilon$
$V(V_{iht})$	$J_t \times I_t$	Inclusive value of a nesting group
$\pi(\pi_{jt})$	$N \times 1$	Population-normalized gross expected profits
$\pi(\pi_{ft})$	$F_t \times 1$	Population-normalized gross expected profits of a firm in market $t$
$\beta$	$K_1 \times 1$	Demand-side linear parameters
$\beta^{\text{ex}}$	$K_1^{\text{ex}} \times 1$	Parameters in $\beta$ on exogenous product characteristics
$\alpha$	$K_1^{\text{en}} \times 1$	Parameters in $\beta$ on endogenous product characteristics
$\Sigma$	$K_2 \times K_2$	Cholesky root of the covariance matrix for unobserved taste heterogeneity
$\Pi$	$K_2 \times D$	Parameters that measures how agent tastes vary with demographics
$\rho$	$H \times 1$	Parameters that measures within nesting group correlation
$\gamma$	$K_3 \times 1$	Supply-side linear parameters
$\gamma^{\text{ex}}$	$K_3^{\text{ex}} \times 1$	Parameters in $\gamma$ on exogenous product characteristics
$\gamma^{\text{en}}$	$K_3^{\text{en}} \times 1$	Parameters in $\gamma$ on endogenous product characteristics
$\theta$	$P \times 1$	Parameters
$Z_D$	$N \times M_D$	Demand-side instruments
$Z_S$	$N \times M_S$	Supply-side instruments
$W$	$M \times M$	Weighting matrix
$S$	$M \times M$	Moment covariances
$q$	$1 \times 1$	Objective value
$g_D$	$N \times M_D$	Demand-side moments
$g_S$	$N \times M_S$	Supply-side moments
$g_M$	$I \times M_M$	Micro moments
$g(g_{jt})$	$N \times (M_D + M_S)$	Demand- and supply-side moments
$g(g_c)$	$C \times (M_D + M_S)$	Clustered demand- and supply-side moments
$\bar{g}_D$	$M_D \times 1$	Averaged demand-side moments
$\bar{g}_S$	$M_S \times 1$	Averaged supply-side moments
$\bar{g}_M$	$M_M \times 1$	Averaged micro moments
$\bar{g}$	$M \times 1$	Averaged moments
$G$	$M \times P$	Jacobian of the averaged moments with respect to $\theta$
$\epsilon$	$J_t \times J_t$	Elasticities of demand in market $t$
$\mathcal{D}$	$J_t \times J_t$	Diversion ratios in market $t$

Continued on next page

Table 1 – continued from previous page

Symbol	Dimensions	Description
$\mathcal{D}$	$J_t \times J_t$	Long-run diversion ratios in market $t$
$\mathcal{M}$	$N \times 1$	Markups
$\mathcal{E}$	$1 \times 1$	Aggregate elasticity of demand of a market
CS	$1 \times 1$	Population-normalized consumer surplus of a market
HHI	$1 \times 1$	Herfindahl-Hirschman Index of a market

## BACKGROUND

The following sections provide a very brief overview of the BLP model and how it is estimated. This goal is to concisely introduce the notation and terminology used throughout the rest of the documentation. For a more in-depth overview, refer to *Conlon and Gortmaker (2020)*.

### 3.1 The Model

There are  $t = 1, 2, \dots, T$  markets, each with  $j = 1, 2, \dots, J_t$  products produced by  $f = 1, 2, \dots, F_t$  firms, for a total of  $N$  products across all markets. There are  $i = 1, 2, \dots, I_t$  individuals/agents who choose among the  $J_t$  products and an outside good  $j = 0$ . These numbers also represent sets. For example,  $J_t = \{1, 2, \dots, J_t\}$ .

#### 3.1.1 Demand

Observed demand-side product characteristics are contained in the  $N \times K_1$  matrix of linear characteristics,  $X_1$ , and the  $N \times K_2$  matrix of nonlinear characteristics,  $X_2$ , which is typically a subset of  $X_1$ . Unobserved demand-side product characteristics,  $\xi$ , are a  $N \times 1$  vector.

In market  $t$ , observed agent characteristics are a  $I_t \times D$  matrix called demographics,  $d$ . Unobserved agent characteristics are a  $I_t \times K_2$  matrix,  $\nu$ .

The indirect utility of agent  $i$  from purchasing product  $j$  in market  $t$  is

$$U_{ijt} = \underbrace{\delta_{jt} + \mu_{ijt}}_{V_{ijt}} + \epsilon_{ijt}, \quad (3.1)$$

in which the mean utility is, in vector-matrix form,

$$\delta = \underbrace{X_1^{\text{en}} \alpha + X_1^{\text{ex}} \beta^{\text{ex}}}_{X_1 \beta} + \xi. \quad (3.2)$$

The  $K_1 \times 1$  vector of demand-side linear parameters,  $\beta$ , is partitioned into two components:  $\alpha$  is a  $K_1^{\text{en}} \times 1$  vector of parameters on the  $N \times K_1^{\text{en}}$  submatrix of endogenous characteristics,  $X_1^{\text{en}}$ , and  $\beta^{\text{ex}}$  is a  $K_1^{\text{ex}} \times 1$  vector of parameters on the  $N \times K_1^{\text{ex}}$  submatrix of exogenous characteristics,  $X_1^{\text{ex}}$ . Usually,  $X_1^{\text{en}} = p$ , prices, so  $\alpha$  is simply a scalar.

The agent-specific portion of utility in a single market is, in vector-matrix form,

$$\mu = X_2(\Sigma \nu' + \Pi d'). \quad (3.3)$$

The model incorporates both observable (demographics) and unobservable taste heterogeneity through random coefficients. For the unobserved heterogeneity, we let  $\nu$  denote independent draws from the standard normal distribution. These are scaled by a  $K_2 \times K_2$  lower-triangular matrix  $\Sigma$ , which denotes the Cholesky root of the covariance matrix for unobserved taste heterogeneity. The  $K_2 \times D$  matrix  $\Pi$  measures how agent tastes vary with demographics.

In the above expression, random coefficients are assumed to be normally distributed. To incorporate one or more lognormal random coefficients, the associated columns in the parenthesized expression can be exponentiated before being pre-multiplied by  $X_2$ . For example, this allows for the coefficient on price to be lognormal so that demand slopes down for all agents. For lognormal random coefficients, a constant column is typically included in  $d$  so that its coefficients in  $\Pi$  parametrize the means of the logs of the random coefficients.

Random idiosyncratic preferences,  $\epsilon_{ijt}$ , are assumed to be Type I Extreme Value, so that conditional on the heterogeneous coefficients, market shares follow the well-known logit form. Aggregate market shares are obtained by integrating over the distribution of individual heterogeneity. They are approximated with Monte Carlo integration or quadrature rules defined by the  $I_t \times K_2$  matrix of integration nodes,  $\nu$ , and an  $I_t \times 1$  vector of integration weights,  $w$ :

$$s_{jt} \approx \sum_{i \in I_t} w_{it} s_{ijt}, \quad (3.4)$$

where the probability that agent  $i$  chooses product  $j$  in market  $t$  is

$$s_{ijt} = \frac{\exp V_{ijt}}{1 + \sum_{k \in J_t} \exp V_{ikt}}. \quad (3.5)$$

There is a one in the denominator because the utility of the outside good is normalized to  $U_{i0t} = 0$ . The scale of utility is normalized by the variance of  $\epsilon_{ijt}$ .

### 3.1.2 Supply

Observed supply-side product characteristics are contained in the  $N \times K_3$  matrix of supply-side characteristics,  $X_3$ . Prices cannot be supply-side characteristics, but non-price product characteristics often overlap with the demand-side characteristics in  $X_1$  and  $X_2$ . Unobserved supply-side product characteristics,  $\omega$ , are a  $N \times 1$  vector.

Firm  $f$  chooses prices in market  $t$  to maximize the profits of its products  $J_{ft} \subset J_t$ :

$$\pi_{ft} = \sum_{j \in J_{ft}} (p_{jt} - c_{jt}) s_{jt}. \quad (3.6)$$

In a single market, the corresponding multi-product differentiated Bertrand first order conditions are, in vector-matrix form,

$$p - c = \underbrace{\Delta^{-1}}_{\eta} s, \quad (3.7)$$

where the multi-product Bertrand markup  $\eta$  depends on  $\Delta$ , a  $J_t \times J_t$  matrix of intra-firm (negative) demand derivatives:

$$\Delta = -\mathcal{H} \odot \frac{\partial s}{\partial p}. \quad (3.8)$$

Here,  $\mathcal{H}$  denotes the market-level ownership or product holdings matrix in the market, where  $\mathcal{H}_{jk}$  is typically 1 if the same firm produces products  $j$  and  $k$ , and 0 otherwise.

To include a supply side, we must specify a functional form for marginal costs:

$$\tilde{c} = f(c) = X_3 \gamma + \omega. \quad (3.9)$$

The most common choices are  $f(c) = c$  and  $f(c) = \log(c)$ .

## 3.2 Estimation

A demand side is always estimated but including a supply side is optional. With only a demand side, there are three sets of parameters to be estimated:  $\beta$  (which may include  $\alpha$ ),  $\Sigma$  and  $\Pi$ . With a supply side, there is also  $\gamma$ . The linear



parameters,  $\beta$  and  $\gamma$ , are typically concentrated out of the problem. The exception is  $\alpha$ , which cannot be concentrated out when there is a supply side because it is needed to compute demand derivatives and hence marginal costs. Linear parameters that are not concentrated out along with unknown nonlinear parameters in  $\Sigma$  and  $\Pi$  are collectively denoted  $\theta$ .

The GMM problem is

$$\min_{\theta} q(\theta) = \bar{g}(\theta)' W \bar{g}(\theta), \quad (3.10)$$

in which  $q(\theta)$  is the GMM objective. By default, PyBLP scales this value by  $N$  so that objectives across different problem sizes are comparable. This behavior can be disabled. In some of the BLP literature and in earlier versions of this package, the objective was scaled by  $N^2$ .

Here,  $W$  is a  $M \times M$  weighting matrix and  $\bar{g}$  is a  $M \times 1$  vector of averaged demand- and supply-side moments:

$$\bar{g} = \begin{bmatrix} \bar{g}_D \\ \bar{g}_S \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \sum_{j,t} Z'_{D,jt} \xi_{jt} \\ \sum_{j,t} Z'_{S,jt} \omega_{jt} \end{bmatrix} \quad (3.11)$$

where  $Z_D$  and  $Z_S$  are  $N \times M_D$  and  $N \times M_S$  matrices of demand- and supply-side instruments.

The vector  $\bar{g}$  contains sample analogues of the demand- and supply-side moment conditions  $E[g_{D,jt}] = E[g_{S,jt}] = 0$  where

$$\begin{bmatrix} g_{D,jt} & g_{S,jt} \end{bmatrix} = \begin{bmatrix} \xi_{jt} Z_{D,jt} & \omega_{jt} Z_{S,jt} \end{bmatrix}. \quad (3.12)$$

In each GMM stage, a nonlinear optimizer finds the  $\hat{\theta}$  that minimizes the GMM objective value  $q(\theta)$ .

### 3.2.1 The Objective

Given a  $\theta$ , the first step to computing the objective  $q(\theta)$  is to compute  $\delta(\theta)$  in each market with the following standard contraction:

$$\delta_{jt} \leftarrow \delta_{jt} + \log s_{jt} - \log s_{jt}(\delta, \theta) \quad (3.13)$$

where  $s$  are the market's observed shares and  $s(\delta, \theta)$  are calculated market shares. Iteration terminates when the norm of the change in  $\delta(\theta)$  is less than a small number.

With a supply side, marginal costs are then computed according to (3.7):

$$c_{jt}(\theta) = p_{jt} - \eta_{jt}(\theta). \quad (3.14)$$

Concentrated out linear parameters are recovered with linear IV-GMM:

$$\begin{bmatrix} \hat{\beta}^{\text{ex}} \\ \hat{\gamma} \end{bmatrix} = (X' Z W Z' X)^{-1} X' Z W Z' Y(\theta) \quad (3.15)$$

where

$$X = \begin{bmatrix} X_1^{\text{ex}} & 0 \\ 0 & X_3 \end{bmatrix}, \quad Z = \begin{bmatrix} Z_D & 0 \\ 0 & Z_S \end{bmatrix}, \quad Y(\theta) = \begin{bmatrix} \delta(\theta) - X_1^{\text{en}} \hat{\alpha} & 0 \\ 0 & \tilde{c}(\theta) \end{bmatrix}. \quad (3.16)$$

With only a demand side,  $\alpha$  can be concentrated out, so  $X = X_1$ ,  $Z = Z_D$ , and  $Y = \delta(\theta)$  recover the full  $\hat{\beta}$  in (3.15).

Finally, the unobserved product characteristics (i.e., the structural errors),

$$\begin{bmatrix} \xi(\theta) \\ \omega(\theta) \end{bmatrix} = \begin{bmatrix} \delta(\theta) - X_1 \hat{\beta} \\ \tilde{c}(\theta) - X_3 \hat{\gamma} \end{bmatrix}, \quad (3.17)$$

are interacted with the instruments to form  $\bar{g}(\theta)$  in (3.11), which gives the GMM objective  $q(\theta)$  in (3.10).

### 3.2.2 The Gradient

The gradient of the GMM objective in (3.10) is

$$\nabla q(\theta) = 2\bar{G}(\theta)'W\bar{g}(\theta) \quad (3.18)$$

where

$$\bar{G} = \begin{bmatrix} \bar{G}_D \\ \bar{G}_S \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \sum_{j,t} Z'_{D,jt} \frac{\partial \xi_{jt}}{\partial \theta} \\ \sum_{j,t} Z'_{S,jt} \frac{\partial \omega_{jt}}{\partial \theta} \end{bmatrix}. \quad (3.19)$$

Writing  $\delta$  as an implicit function of  $s$  in (3.4) gives the demand-side Jacobian:

$$\frac{\partial \xi}{\partial \theta} = \frac{\partial \delta}{\partial \theta} = - \left( \frac{\partial s}{\partial \delta} \right)^{-1} \frac{\partial s}{\partial \theta}. \quad (3.20)$$

The supply-side Jacobian is derived from the definition of  $\tilde{c}$  in (3.9):

$$\frac{\partial \omega}{\partial \theta} = \frac{\partial \tilde{c}}{\partial \theta_p} = - \frac{\partial \tilde{c}}{\partial c} \frac{\partial \eta}{\partial \theta}. \quad (3.21)$$

The second term in this expression is derived from the definition of  $\eta$  in (3.7):

$$\frac{\partial \eta}{\partial \theta} = -\Delta^{-1} \left( \frac{\partial \Delta}{\partial \theta} \eta + \frac{\partial \Delta}{\partial \xi} \eta \frac{\partial \xi}{\partial \theta} \right). \quad (3.22)$$

### 3.2.3 Weighting Matrices

Conventionally, the 2SLS weighting matrix is used in the first stage:

$$W = \begin{bmatrix} (Z'_D Z_D / N)^{-1} & 0 \\ 0 & (Z'_S Z_S / N)^{-1} \end{bmatrix}. \quad (3.23)$$

With two-step GMM,  $W$  is updated before the second stage according to

$$W = S^{-1}. \quad (3.24)$$

For heteroscedasticity robust weighting matrices,

$$S = \frac{1}{N} \sum_{j,t} g_{jt} g'_{jt}. \quad (3.25)$$

For clustered weighting matrices with  $c = 1, 2, \dots, C$  clusters,

$$S = \frac{1}{N} \sum_{c=1}^C g_c g'_c, \quad (3.26)$$

where, letting the set  $J_{ct} \subset J_t$  denote products in cluster  $c$  and market  $t$ ,

$$g_c = \sum_{t \in T} \sum_{j \in J_{ct}} g_{jt}. \quad (3.27)$$

For unadjusted weighting matrices,

$$S = \frac{1}{N} \begin{bmatrix} \sigma_\xi^2 Z'_D Z_D & \sigma_{\xi\omega} Z'_D Z_S \\ \sigma_{\xi\omega} Z'_S Z_D & \sigma_\omega^2 Z'_S Z_S \end{bmatrix} \quad (3.28)$$

where

$$\text{Var}(\xi, \omega) = \begin{bmatrix} \sigma_\xi^2 & \sigma_{\xi\omega} \\ \sigma_{\xi\omega} & \sigma_\omega^2 \end{bmatrix}. \quad (3.29)$$

### 3.2.4 Standard Errors

The covariance matrix of the estimated parameters is

$$\text{Var}(\hat{\theta}) = (\bar{G}'W\bar{G})^{-1}\bar{G}'WSW\bar{G}(\bar{G}'W\bar{G})^{-1}. \quad (3.30)$$

Standard errors are the square root of the diagonal of this matrix divided by  $N$ .

If the weighting matrix was chosen such that  $W = S^{-1}$ , this simplifies to

$$\text{Var}(\hat{\theta}) = (\bar{G}'W\bar{G})^{-1}. \quad (3.31)$$

Standard errors extracted from this simpler expression are called unadjusted.

### 3.3 Fixed Effects

The unobserved product characteristics can be partitioned into

$$\begin{bmatrix} \xi_{jt} \\ \omega_{jt} \end{bmatrix} = \begin{bmatrix} \xi_{k_1} + \xi_{k_2} + \dots + \xi_{k_{E_D}} + \Delta\xi_{jt} \\ \omega_{\ell_1} + \omega_{\ell_2} + \dots + \omega_{\ell_{E_S}} + \Delta\omega_{jt} \end{bmatrix} \quad (3.32)$$

where  $k_1, k_2, \dots, k_{E_D}$  and  $\ell_1, \ell_2, \dots, \ell_{E_S}$  index unobserved characteristics that are fixed across  $E_D$  and  $E_S$  dimensions. For example, with  $E_D = 1$  dimension of product fixed effects,  $\xi_{jt} = \xi_j + \Delta\xi_{jt}$ .

Small numbers of fixed effects can be estimated with dummy variables in  $X_1, X_3, Z_D$ , and  $Z_S$ . However, this approach does not scale with high dimensional fixed effects because it requires constructing and inverting an infeasibly large matrix in (3.15).

Instead, fixed effects are typically absorbed into  $X, Z$ , and  $Y(\theta)$  in (3.15). With one fixed effect, these matrices are simply de-measured within each level of the fixed effect. Both  $X$  and  $Z$  can be de-measured just once, but  $Y(\theta)$  must be de-measured for each new  $\theta$ .

This procedure is equivalent to replacing each column of the matrices with residuals from a regression of the column on the fixed effect. The Frish-Waugh-Lovell (FWL) theorem of *Frisch and Waugh (1933)* and *Lovell (1963)* guarantees that using these residualized matrices gives the same results as including fixed effects as dummy variables. When  $E_D > 1$  or  $E_S > 1$ , the matrices are residualized with more involved algorithms.

Once fixed effects have been absorbed, estimation is as described above with the structural errors  $\Delta\xi$  and  $\Delta\omega$ .

### 3.4 Micro Moments

In the spirit of *Imbens and Lancaster (1994)*, *Petrin (2002)*, and *Berry, Levinsohn, and Pakes (2004)*, more detailed micro data on individual agent decisions can be used to supplement the standard demand- and supply-side moments  $\bar{g}_D$  and  $\bar{g}_S$  in (3.11) with an additional  $m = 1, 2, \dots, M_M$  averaged micro moments,  $\bar{g}_M$ , for a total of  $M = M_D + M_S + M_M$  averaged moments:

$$\bar{g} = \begin{bmatrix} \bar{g}_D \\ \bar{g}_S \\ \bar{g}_M \end{bmatrix}. \quad (3.33)$$

Each micro moment  $m$  is approximated in a set  $T_m \subset T$  of markets in which its micro data are relevant and then averaged across these markets:

$$\bar{g}_{M,m} \approx \frac{1}{T_m} \sum_{t \in T_m} g_{M,mt}. \quad (3.34)$$

The vector  $\bar{g}_M$  contains sample analogues of micro moment conditions  $E[g_{M,mt}] = 0$  where  $g_{M,mt}$  is typically a function of choice probabilities, data in market  $t$ , and a statistic  $\mathcal{V}_{mt}$  computed from survey data that the moment aims to match.

Mico moments are computed for each  $\theta$  and contribute to the GMM objective  $q(\theta)$  in (3.10). Their derivatives with respect to  $\theta$  are added as rows to  $\bar{G}$  in (3.19), and blocks are added to both  $W$  and  $S$  in (3.23) and (3.24). The covariance between standard moments and micro moments is assumed to be zero, so these matrices are block-diagonal. The covariance between micro moments  $m$  and  $n$  in  $S$  is set to zero if  $T_{mn} = T_m \cap T_n = \emptyset$  and otherwise is

$$\text{Cov}(\bar{g}_{M,m}, \bar{g}_{M,n}) = \frac{1}{T_m \times T_n} \sum_{t \in T_{mn}} \text{Cov}(g_{M,mt}, g_{M,nt}). \quad (3.35)$$

### 3.5 Random Coefficients Nested Logit

Incorporating parameters that measure within nesting group correlation gives the random coefficients nested logit (RCNL) model of *Brenkers and Verboven (2006)* and *Grigolon and Verboven (2014)*. There are  $h = 1, 2, \dots, H$  nesting groups and each product  $j$  is assigned to a group  $h(j)$ . The set  $J_{ht} \subset J_t$  denotes the products in group  $h$  and market  $t$ .

In the RCNL model, idiosyncratic preferences are partitioned into

$$\epsilon_{ijt} = \bar{\epsilon}_{ih(j)t} + (1 - \rho_{h(j)})\bar{\epsilon}_{ijt} \quad (3.36)$$

where  $\bar{\epsilon}_{ijt}$  is Type I Extreme Value and  $\bar{\epsilon}_{ih(j)t}$  is distributed such that  $\epsilon_{ijt}$  is still Type I Extreme Value.

The nesting parameters,  $\rho$ , can either be a  $H \times 1$  vector or a scalar so that for all groups  $\rho_h = \rho$ . Letting  $\rho \rightarrow 0$  gives the standard BLP model and  $\rho \rightarrow 1$  gives division by zero errors. With  $\rho_h \in (0, 1)$ , the expression for choice probabilities in (3.5) becomes more complicated:

$$s_{ijt} = \frac{\exp[V_{ijt}/(1 - \rho_{h(j)})]}{\exp[V_{ih(j)t}/(1 - \rho_{h(j)})]} \cdot \frac{\exp V_{ih(j)t}}{1 + \sum_{h \in H} \exp V_{iht}} \quad (3.37)$$

where

$$V_{iht} = (1 - \rho_h) \log \sum_{k \in J_{ht}} \exp[V_{ikt}/(1 - \rho_h)]. \quad (3.38)$$

The contraction for  $\delta(\theta)$  in (3.13) is also slightly different:

$$\delta_{jt} \leftarrow \delta_{jt} + (1 - \rho_{h(j)})[\log s_{jt} - \log s_{jt}(\delta, \theta)]. \quad (3.39)$$

Otherwise, estimation is as described above with  $\rho$  included in  $\theta$ .

### 3.6 Logit and Nested Logit

Letting  $\Sigma = 0$  gives the simpler logit (or nested logit) model where there is a closed-form solution for  $\delta$ . In the logit model,

$$\delta_{jt} = \log s_{jt} - \log s_{0t}, \quad (3.40)$$

and a lack of nonlinear parameters means that nonlinear optimization is often unneeded.

In the nested logit model,  $\rho$  must be optimized over, but there is still a closed-form solution for  $\delta$ :

$$\delta_{jt} = \log s_{jt} - \log s_{0t} - \rho_{h(j)}[\log s_{jt} - \log s_{h(j)t}]. \quad (3.41)$$

where

$$s_{ht} = \sum_{j \in J_{ht}} s_{jt}. \quad (3.42)$$

In both models, a supply side can still be estimated jointly with demand. Estimation is as described above with a representative agent in each market:  $I_t = 1$  and  $w_1 = 1$ .

### 3.7 Equilibrium Prices

Counterfactual evaluation, synthetic data simulation, and optimal instrument generation often involve solving for prices implied by the Bertrand first order conditions in (3.7). Solving this system with Newton's method is slow and iterating over  $p \leftarrow c + \eta(p)$  may not converge because it is not a contraction.

Instead, *Morrow and Skerlos (2011)* reformulate the solution to (3.7):

$$p - c = \underbrace{\Lambda^{-1}(\mathcal{H} \odot \Gamma)'(p - c) - \Lambda^{-1}s}_{\zeta} \quad (3.43)$$

where  $\Lambda$  is a diagonal  $J_t \times J_t$  matrix approximated by

$$\Lambda_{jj} \approx \sum_{i \in I_t} w_{it} s_{ijt} \frac{\partial U_{ijt}}{\partial p_{jt}} \quad (3.44)$$

and  $\Gamma$  is a dense  $J_t \times J_t$  matrix approximated by

$$\Gamma_{jk} \approx \sum_{i \in I_t} w_{it} s_{ijt} s_{ikt} \frac{\partial U_{ijt}}{\partial p_{jt}}. \quad (3.45)$$

Equilibrium prices are computed by iterating over the  $\zeta$ -markup equation in (3.43),

$$p \leftarrow c + \zeta(p), \quad (3.46)$$

which, unlike (3.7), is a contraction. Iteration terminates when the norm of firms' first order conditions,  $\|\Lambda(p)(p - c - \zeta(p))\|$ , is less than a small number.

If marginal costs depend on quantity, then they also depend on prices and need to be updated during each iteration:  $c_{jt} = c_{jt}(s_{jt}(p))$ .



## TUTORIAL

This section uses a series of [Jupyter Notebooks](#) to explain how PyBLP can be used to solve example problems, compute post-estimation outputs, and simulate problems. Other examples can be found in the [API Documentation](#).

---

The [online version](#) of the following section may be easier to read.

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## 4.1 Logit and Nested Logit Tutorial

```
import pyblp
import numpy as np
import pandas as pd

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

In this tutorial, we'll use data from *Nevo (2000)* to solve the paper's fake cereal problem. Locations of CSV files that contain the data are in the `data` module.

We will compare two simple models, the plain (IIA) logit model and the nested logit (GEV) model using the fake cereal dataset of *Nevo (2000)*.

### 4.1.1 Theory of Plain Logit

Let's start with the plain logit model under independence of irrelevant alternatives (IIA). In this model (indirect) utility is given by

$$U_{ijt} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt} + \epsilon_{ijt}, \quad (4.1)$$

where  $\epsilon_{ijt}$  is distributed IID with the Type I Extreme Value (Gumbel) distribution. It is common to normalize the mean utility of the outside good to zero so that  $U_{i0t} = \epsilon_{i0t}$ . This gives us aggregate market shares

$$s_{jt} = \frac{\exp(\alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt})}{1 + \sum_k \exp(\alpha p_{kt} + x_{kt} \beta^{\text{ex}} + \xi_{kt})}. \quad (4.2)$$

If we take logs we get

$$\log s_{jt} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt} - \log \sum_k \exp(\alpha p_{kt} + x_{kt} \beta^{\text{ex}} + \xi_{kt}) \quad (4.3)$$

and

$$\log s_{0t} = -\log \sum_k \exp(\alpha p_{kt} + x_{kt} \beta^{\text{ex}} + \xi_{kt}). \quad (4.4)$$



By differencing the above we get a linear estimating equation:

$$\log s_{jt} - \log s_{0t} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt}. \quad (4.5)$$

Because the left hand side is data, we can estimate this model using linear IV GMM.

### 4.1.2 Application of Plain Logit

A Logit *Problem* can be created by simply excluding the formulation for the nonlinear parameters,  $X_2$ , along with any agent information. In other words, it requires only specifying the *linear component* of demand.

We'll set up and solve a simple version of the fake data cereal problem from *Nevo (2000)*. Since we won't include any demand-side nonlinear characteristics or parameters, we don't have to worry about configuring an optimization routine.

There are some important reserved variable names:

- `market_ids` are the unique market identifiers which we subscript with  $t$ .
- `shares` specifies the market shares which need to be between zero and one, and within a market ID,  $\sum_j s_{jt} \leq 1$ .
- `prices` are prices  $p_{jt}$ . These have some special properties and are *always* treated as endogenous.
- `demand_instruments0`, `demand_instruments1`, and so on are numbered demand instruments. These represent only the *excluded* instruments. The exogenous regressors in  $X_1$  will be automatically added to the set of instruments.

We begin with two steps:

1. Load the product data which at a minimum consists of `market_ids`, `shares`, `prices`, and at least a single column of demand instruments, `demand_instruments0`.
2. Define a *Formulation* for the  $X_1$  (linear) demand model.
  - This and all other formulas are similar to R and `patsy` formulas.
  - It includes a constant by default. To exclude the constant, specify either a 0 or a -1.
  - To efficiently include fixed effects, use the `absorb` option and specify which categorical variables you would like to absorb.
  - Some model reduction may happen automatically. The constant will be excluded if you include fixed effects and some precautions are taken against collinearity. However, you will have to make sure that differently-named variables are not collinear.
3. Combine the *Formulation* and product data to construct a *Problem*.
4. Use `Problem.solve` to estimate parameters.

## Loading the Data

The `product_data` argument of *Problem* should be a structured array-like object with fields that store data. Product data can be a structured NumPy array, a pandas DataFrame, or other similar objects.

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
product_data.head()
```

```

 market_ids  city_ids  quarter  product_ids  firm_ids  brand_ids  shares  \
0      C01Q1         1         1      F1B04         1         4  0.012417
1      C01Q1         1         1      F1B06         1         6  0.007809
2      C01Q1         1         1      F1B07         1         7  0.012995
3      C01Q1         1         1      F1B09         1         9  0.005770
4      C01Q1         1         1      F1B11         1        11  0.017934

   prices  sugar  mushy  ...  demand_instruments10  demand_instruments11  \
0  0.072088     2     1  ...           2.116358           -0.154708
1  0.114178    18     1  ...           -7.374091           -0.576412
2  0.132391     4     1  ...           2.187872           -0.207346
3  0.130344     3     0  ...           2.704576            0.040748
4  0.154823    12     0  ...           1.261242            0.034836

 demand_instruments12  demand_instruments13  demand_instruments14  \
0          -0.005796           0.014538           0.126244
1           0.012991           0.076143           0.029736
2           0.003509           0.091781           0.163773
3          -0.003724           0.094732           0.135274
4          -0.000568           0.102451           0.130640

 demand_instruments15  demand_instruments16  demand_instruments17  \
0           0.067345           0.068423           0.034800
1           0.087867           0.110501           0.087784
2           0.111881           0.108226           0.086439
3           0.088090           0.101767           0.101777
4           0.084818           0.101075           0.125169

 demand_instruments18  demand_instruments19
0           0.126346           0.035484
1           0.049872           0.072579
2           0.122347           0.101842
3           0.110741           0.104332
4           0.133464           0.121111

```

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```
[5 rows x 30 columns]
```

The product data contains `market_ids`, `product_ids`, `firm_ids`, `shares`, `prices`, a number of other IDs and product characteristics, and some pre-computed excluded `demand_instruments0`, `demand_instruments1`, and so on. The `product_ids` will be incorporated as fixed effects.

For more information about the instruments and the example data as a whole, refer to the [data](#) module.

### Setting Up the Problem

We can combine the *Formulation* and `product_data` to construct a *Problem*. We pass the *Formulation* first and the `product_data` second. We can also display the properties of the problem by typing its name.

```
logit_formulation = pyblp.Formulation('prices', absorb='C(product_ids)')
logit_formulation
prices + Absorb[C(product_ids)]
```

```
problem = pyblp.Problem(logit_formulation, product_data)
problem
```

Dimensions:

```
=====
  T   N   F   K1  MD  ED
  ---
94  2256  5   1   20  1
=====
```

Formulations:

```
=====
      Column Indices:          0
-----
X1: Linear Characteristics prices
=====
```

Two sets of properties are displayed:

1. Dimensions of the data.
2. Formulations of the problem.

The dimensions describe the shapes of matrices as laid out in *Notation*. They include:

- $T$  is the number of markets.
- $N$  is the length of the dataset (the number of products across all markets).
- $F$  is the number of firms, which we won't use in this example.
- $K_1$  is the dimension of the linear demand parameters.
- $M_D$  is the dimension of the instrument variables (excluded instruments and exogenous regressors).
- $E_D$  is the number of fixed effect dimensions (one-dimensional fixed effects, two-dimensional fixed effects, etc.).

There is only a single *Formulation* for this model.

- $X_1$  is the linear component of utility for demand and depends only on prices (after the fixed effects are removed).

## Solving the Problem

The `Problem.solve` method always returns a `ProblemResults` class, which can be used to compute post-estimation outputs. See the *post estimation* tutorial for more information.

```
logit_results = problem.solve()
logit_results
```

Problem Results Summary:

```
=====
GMM   Objective  Clipped  Weighting Matrix
Step   Value      Shares   Condition Number
-----
  2    +1.9E+02    0        +5.7E+07
=====
```

Cumulative Statistics:

```
=====
Computation  Objective
Time         Evaluations
-----
00:00:00    2
=====
```

Beta Estimates (Robust SEs in Parentheses):

```
=====
prices
```

(continues on next page)

```
-----
-3.0E+01
(+1.0E+00)
=====
```

### 4.1.3 Theory of Nested Logit

We can extend the logit model to allow for correlation within a group  $h$  so that

$$U_{ijt} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt} + \bar{\epsilon}_{h(j)ti} + (1 - \rho) \bar{\epsilon}_{ijt}. \quad (4.6)$$

Now, we require that  $\epsilon_{jti} = \bar{\epsilon}_{h(j)ti} + (1 - \rho) \bar{\epsilon}_{ijt}$  is distributed IID with the Type I Extreme Value (Gumbel) distribution. As  $\rho \rightarrow 1$ , all consumers stay within their group. As  $\rho \rightarrow 0$ , this collapses to the IIA logit. Note that if we wanted, we could allow  $\rho$  to differ between groups with the notation  $\rho_{h(j)}$ .

This gives us aggregate market shares as the product of two logits, the within group logit and the across group logit:

$$s_{jt} = \frac{\exp[V_{jt}/(1 - \rho)]}{\exp[V_{h(j)t}/(1 - \rho)]} \cdot \frac{\exp V_{h(j)t}}{1 + \sum_h \exp V_{ht}}, \quad (4.7)$$

where  $V_{jt} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt}$ .

After some work we again obtain the linear estimating equation:

$$\log s_{jt} - \log s_{0t} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \rho \log s_{j|h(j)t} + \xi_{jt}, \quad (4.8)$$

where  $s_{j|h(j)t} = s_{jt}/s_{h(j)t}$  and  $s_{h(j)t}$  is the share of group  $h$  in market  $t$ . See *Berry (1994)* or *Cardell (1997)* for more information.

Again, the left hand side is data, though the  $\ln s_{j|h(j)t}$  is clearly endogenous which means we must instrument for it. Rather than include  $\ln s_{j|h(j)t}$  along with the linear components of utility,  $X_1$ , whenever `nesting_ids` are included in `product_data`,  $\rho$  is treated as a nonlinear  $X_2$  parameter. This means that the linear component is given instead by

$$\log s_{jt} - \log s_{0t} - \rho \log s_{j|h(j)t} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt}. \quad (4.9)$$

This is done for two reasons:

1. It forces the user to treat  $\rho$  as an endogenous parameter.
2. It extends much more easily to the RCNL model of *Brenkers and Verboven (2006)*.

A common choice for an additional instrument is the number of products per nest.

## 4.1.4 Application of Nested Logit

By including `nesting_ids` (another reserved name) as a field in `product_data`, we tell the package to estimate a nested logit model, and we don't need to change any of the formulas. We show how to construct the category groupings in two different ways:

1. We put all products in a single nest (only the outside good in the other nest).
2. We put products into two nests (either mushy or non-mushy).

We also construct an additional instrument based on the number of products per nest. Typically this is useful as a source of exogenous variation in the within group share  $\ln s_{j|h(j)t}$ . However, in this example because the number of products per nest does not vary across markets, if we include product fixed effects, this instrument is irrelevant.

We'll define a function that constructs the additional instrument and solves the nested logit problem. We'll exclude product ID fixed effects, which are collinear with `mushy`, and we'll choose  $\rho = 0.7$  as the initial value at which the optimization routine will start.

```
def solve_nl(df):
    groups = df.groupby(['market_ids', 'nesting_ids'])
    df['demand_instruments20'] = groups['shares'].transform(np.size)
    nl_formulation = pyblp.Formulation('0 + prices')
    problem = pyblp.Problem(nl_formulation, df)
    return problem.solve(rho=0.7)
```

First, we'll solve the problem when there's a single nest for all products, with the outside good in its own nest.

```
df1 = product_data.copy()
df1['nesting_ids'] = 1
nl_results1 = solve_nl(df1)
nl_results1
```

Problem Results Summary:

```
=====
GMM   Objective   Projected   Reduced   Clipped   Weighting Matrix   Covariance Matrix
Step   Value         Gradient   Norm      Hessian   Shares            Condition Number   Condition Number
-----
  2    +2.0E+02     +7.9E-10   +1.1E+04   0          +2.0E+09           +3.0E+04
=====
```

Cumulative Statistics:

```
=====
Computation   Optimizer   Optimization   Objective
Time          Converged   Iterations     Evaluations
-----
```

(continues on next page)

```

00:00:05      Yes          3          8
=====

Rho Estimates (Robust SEs in Parentheses):
=====
All Groups
-----
+9.8E-01
(+1.4E-02)
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-1.2E+00
(+4.0E-01)
=====

```

When we inspect the *Problem*, the only changes from the plain logit model is the additional instrument that contributes to  $M_D$  and the inclusion of  $H$ , the number of nesting categories.

```

nl_results1.problem
Dimensions:
=====
  T      N      F      K1      MD      H
  ---      ---      ---      ---      ---      ---
 94     2256     5       1       21       1
=====

Formulations:
=====
      Column Indices:          0
-----
X1: Linear Characteristics prices
=====

```

Next, we'll solve the problem when there are two nests for mushy and non-mushy.

```

df2 = product_data.copy()
df2['nesting_ids'] = df2['mushy']
nl_results2 = solve_n1(df2)
nl_results2

Problem Results Summary:
=====
GMM   Objective   Projected   Reduced   Clipped   Weighting Matrix   Covariance Matrix
Step  Value       Gradient Norm Hessian   Shares   Condition Number   Condition Number
-----
  2    +6.9E+02    +8.2E-09    +5.6E+03    0        +5.1E+08           +2.0E+04
=====

Cumulative Statistics:
=====
Computation   Optimizer   Optimization   Objective
  Time       Converged   Iterations     Evaluations
-----
 00:00:06      Yes         4              13
=====

Rho Estimates (Robust SEs in Parentheses):
=====
All Groups
-----
+8.9E-01
(+1.9E-02)
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-7.8E+00
(+4.8E-01)
=====

```

For both cases we find that  $\hat{\rho} > 0.8$ .  
 Finally, we'll also look at the adjusted parameter on prices,  $\alpha/(1 - \rho)$ .



```
nl_results1.beta[0] / (1 - nl_results1.rho)
array([[ -67.39338888]])
```

```
nl_results2.beta[0] / (1 - nl_results2.rho)
array([[ -72.27074638]])
```

### Treating Within Group Shares as Exogenous

The package is designed to prevent the user from treating the within group share,  $\log s_{j|h(j)t}$ , as an exogenous variable. For example, if we were to compute a `group_share` variable and use the algebraic functionality of *Formulation* by including the expression `log(shares / group_share)` in our formula for  $X_1$ , the package would raise an error because the package knows that `shares` should not be included in this formulation.

To demonstrate why this is a bad idea, we'll override this feature by calculating  $\log s_{j|h(j)t}$  and including it as an additional variable in  $X_1$ . To do so, we'll first re-define our function for setting up and solving the nested logit problem.

```
def solve_nl2(df):
    groups = df.groupby(['market_ids', 'nesting_ids'])
    df['group_share'] = groups['shares'].transform(np.sum)
    df['within_share'] = df['shares'] / df['group_share']
    df['demand_instruments20'] = groups['shares'].transform(np.size)
    nl2_formulation = pyblp.Formulation('0 + prices + log(within_share)')
    problem = pyblp.Problem(nl2_formulation, df.drop(columns=['nesting_ids']))
    return problem.solve()
```

Again, we'll solve the problem when there's a single nest for all products, with the outside good in its own nest.

```
nl2_results1 = solve_nl2(df1)
nl2_results1
```

Problem Results Summary:

```
=====
GMM   Objective  Clipped  Weighting Matrix  Covariance Matrix
Step   Value      Shares   Condition Number  Condition Number
-----
  2    +2.0E+02     0        +2.1E+09          +1.1E+04
=====
```

Cumulative Statistics:

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```

=====
Computation   Objective
   Time      Evaluations
-----
00:00:00      2
=====

```

Beta Estimates (Robust SEs in Parentheses):

```

=====
prices      log(within_share)
-----
-1.0E+00    +9.9E-01
(+2.4E-01)  (+7.9E-03)
=====

```

And again, we'll solve the problem when there are two nests for mushy and non-mushy.

```

nl2_results2 = solve_nl2(df2)
nl2_results2

```

Problem Results Summary:

```

=====
GMM   Objective   Clipped   Weighting Matrix   Covariance Matrix
Step   Value        Shares    Condition Number   Condition Number
-----
2     +7.0E+02      0         +5.5E+08           +7.7E+03
=====

```

Cumulative Statistics:

```

=====
Computation   Objective
   Time      Evaluations
-----
00:00:00      2
=====

```

Beta Estimates (Robust SEs in Parentheses):

```

=====
prices      log(within_share)
-----
-6.8E+00    +9.3E-01
=====

```

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```
(+2.9E-01)      (+1.1E-02)
=====
```

One can observe that we obtain parameter estimates which are quite different than above.

```
n12_results1.beta[0] / (1 - n12_results1.beta[1])
```

```
array([-86.37368446])
```

```
n12_results2.beta[0] / (1 - n12_results2.beta[1])
```

```
array([-100.14496891])
```

---

The [online version](#) of the following section may be easier to read.

---

## 4.2 Random Coefficients Logit Tutorial with the Fake Cereal Data

```
import pyblp
import numpy as np
import pandas as pd

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

In this tutorial, we'll use data from *Nevo (2000)* to solve the paper's fake cereal problem. Locations of CSV files that contain the data are in the `data` module.

### 4.2.1 Theory of Random Coefficients Logit

The random coefficients model extends the plain logit model by allowing for correlated tastes for different product characteristics. In this model (indirect) utility is given by

$$u_{ijt} = \alpha_i p_{jt} + x_{jt} \beta_i^{\text{ex}} + \xi_{jt} + \epsilon_{ijt} \quad (4.10)$$

The main addition is that  $\beta_i = (\alpha_i, \beta_i^{\text{ex}})$  have individual specific subscripts  $i$ .

Conditional on  $\beta_i$ , the individual market share follow the same logit form as before. But now we must integrate over heterogeneous individuals to get the aggregate market share:

$$s_{jt}(\alpha, \beta, \theta) = \int \frac{\exp(\alpha_i p_{jt} + x_{jt} \beta_i^{\text{ex}} + \xi_{jt})}{1 + \sum_k \exp(\alpha_i p_{jt} + x_{kt} \beta_i^{\text{ex}} + \xi_{kt})} f(\alpha_i, \beta_i | \theta). \quad (4.11)$$

In general, this integral needs to be calculated numerically. This also means that we can't directly linearize the model. It is common to re-parametrize the model to separate the aspects of mean utility that all individuals agree on,  $\delta_{jt} = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt}$ , from the individual specific heterogeneity,  $\mu_{ijt}(\theta)$ . This gives us

$$s_{jt}(\delta_{jt}, \theta) = \int \frac{\exp(\delta_{jt} + \mu_{ijt})}{1 + \sum_k \exp(\delta_{kt} + \mu_{ikt})} f(\mu_{it} | \theta). \quad (4.12)$$

Given a guess of  $\theta$  we can solve the system of nonlinear equations for the vector  $\delta$  which equates observed and predicted market share  $s_{jt} = s_{jt}(\delta, \theta)$ . Now we can perform a linear IV GMM regression of the form

$$\delta_{jt}(\theta) = \alpha p_{jt} + x_{jt} \beta^{\text{ex}} + \xi_{jt}. \quad (4.13)$$

The moments are constructed by interacting the predicted residuals  $\hat{\xi}_{jt}(\theta)$  with instruments  $z_{jt}$  to form

$$\bar{g}(\theta) = \frac{1}{N} \sum_{j,t} z'_{jt} \hat{\xi}_{jt}(\theta). \quad (4.14)$$

## 4.2.2 Application of Random Coefficients Logit with the Fake Cereal Data

To include random coefficients we need to add a *Formulation* for the demand-side nonlinear characteristics  $X_2$ .

Just like in the logit case we have the same reserved field names in `product_data`:

- `market_ids` are the unique market identifiers which we subscript  $t$ .
- `shares` specifies the market share which need to be between zero and one, and within a market ID,  $\sum_j s_{jt} < 1$ .
- `prices` are prices  $p_{jt}$ . These have some special properties and are *always* treated as endogenous.
- `demand_instruments0`, `demand_instruments1`, and so on are numbered demand instruments. These represent only the *excluded* instruments. The exogenous regressors in  $X_1$  (of which  $X_2$  is typically a subset) will be automatically added to the set of instruments.

We proceed with the following steps:

1. Load the `product_data` which at a minimum consists of `market_ids`, `shares`, `prices`, and at least a single column of demand instruments, `demand_instruments0`.
2. Define a *Formulation* for the  $X_1$  (linear) demand model.
  - This and all other formulas are similar to R and `patsy` formulas.
  - It includes a constant by default. To exclude the constant, specify either a 0 or a `-1`.
  - To efficiently include fixed effects, use the `absorb` option and specify which categorical variables you would like to absorb.
  - Some model reduction may happen automatically. The constant will be excluded if you include fixed effects and some precautions are taken against collinearity. However, you will have to make sure that differently-named variables are not collinear.
3. Define a *Formulation* for the  $X_2$  (nonlinear) demand model.
  - Include only the variables over which we want random coefficients.
  - Do not absorb or include fixed effects.

- It will include a random coefficient on the constant (to capture inside good vs. outside good preference) unless you specify not to with a 0 or a -1.
4. Define an *Integration* configuration to solve the market share integral from several available options:
    - Monte Carlo integration (pseudo-random draws).
    - Product rule quadrature.
    - Sparse grid quadrature.
  5. Combine *Formulation* classes, `product_data`, and the *Integration* configuration to construct a *Problem*.
  6. Use the *Problem.solve* method to estimate parameters.
    - It is required to specify an initial guess of the nonlinear parameters. This serves two primary purposes: speeding up estimation and indicating to the solver through initial values of zero which parameters are restricted to be always zero.

### Specification of Random Taste Parameters

It is common to assume that  $f(\beta_i | \theta)$  follows a multivariate normal distribution, and to break it up into three parts:

1. A mean  $K_1 \times 1$  taste which all individuals agree on,  $\beta$ .
2. A  $K_2 \times K_2$  covariance matrix,  $\Sigma$ .
3. Any  $K_2 \times D$  interactions,  $\Pi$ , with observed  $D \times 1$  demographic data,  $d_i$ .

Together this gives us that

$$\beta_i \sim N(\beta + \Pi d_i, \Sigma). \quad (4.15)$$

*Problem.solve* takes an initial guess  $\Sigma_0$  of  $\Sigma$ . It guarantees that  $\hat{\Sigma}$  (the estimated parameters) will have the same sparsity structure as  $\Sigma_0$ . So any zero element of  $\Sigma$  is restricted to be zero in the solution  $\hat{\Sigma}$ . For example, a popular restriction is that  $\Sigma$  is diagonal, this can be achieved by passing a diagonal matrix as  $\Sigma_0$ .

As is common with multivariate normal distributions,  $\Sigma$  is not estimated directly. Rather, its matrix square (Cholesky) root  $L$  is estimated where  $LL' = \Sigma$ .

### Loading the Data

The `product_data` argument of *Problem* should be a structured array-like object with fields that store data. Product data can be a structured NumPy array, a pandas DataFrame, or other similar objects.

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
product_data.head()
```

```

market_ids  city_ids  quarter  product_ids  firm_ids  brand_ids  shares  \
0  C01Q1      1      1      F1B04      1          4  0.012417
1  C01Q1      1      1      F1B06      1          6  0.007809
2  C01Q1      1      1      F1B07      1          7  0.012995
3  C01Q1      1      1      F1B09      1          9  0.005770
4  C01Q1      1      1      F1B11      1         11  0.017934

prices  sugar  mushy  ...  demand_instruments10  demand_instruments11  \
0  0.072088    2    1  ...          2.116358          -0.154708
1  0.114178   18    1  ...         -7.374091          -0.576412
2  0.132391    4    1  ...          2.187872          -0.207346
3  0.130344    3    0  ...          2.704576           0.040748
4  0.154823   12    0  ...          1.261242           0.034836

demand_instruments12  demand_instruments13  demand_instruments14  \
0          -0.005796          0.014538          0.126244
1           0.012991          0.076143          0.029736
2           0.003509          0.091781          0.163773
3          -0.003724          0.094732          0.135274
4          -0.000568          0.102451          0.130640

demand_instruments15  demand_instruments16  demand_instruments17  \
0           0.067345          0.068423          0.034800
1           0.087867          0.110501          0.087784
2           0.111881          0.108226          0.086439
3           0.088090          0.101767          0.101777
4           0.084818          0.101075          0.125169

demand_instruments18  demand_instruments19
0           0.126346          0.035484
1           0.049872          0.072579
2           0.122347          0.101842
3           0.110741          0.104332
4           0.133464          0.121111

[5 rows x 30 columns]

```

The product data contains `market_ids`, `product_ids`, `firm_ids`, `shares`, `prices`, a number of other firm IDs and product characteristics, and some pre-computed excluded `demand_instruments0`, `demand_instruments1`, and so on. The `product_ids` will be incorporated as fixed effects.

For more information about the instruments and the example data as a whole, refer to the [data](#) module.

## Setting Up and Solving the Problem Without Demographics

Formulations, product data, and an integration configuration are collectively used to initialize a *Problem*. Once initialized, *Problem.solve* runs the estimation routine. The arguments to *Problem.solve* configure how estimation is performed. For example, *optimization* and *iteration* arguments configure the optimization and iteration routines that are used by the outer and inner loops of estimation.

We'll specify *Formulation* configurations for  $X_1$ , the demand-side linear characteristics, and  $X_2$ , the nonlinear characteristics.

- The formulation for  $X_1$  consists of `prices` and fixed effects constructed from `product_ids`, which we will absorb using `absorb` argument of *Formulation*.
- If we were interested in reporting estimates for each fixed effect, we could replace the formulation for  $X_1$  with `Formulation('prices + C(product_ids)')`.
- Because `sugar`, `mushy`, and the constant are collinear with `product_ids`, we can include them in  $X_2$  but not in  $X_1$ .

```
X1_formulation = pyblp.Formulation('0 + prices', absorb='C(product_ids)')
X2_formulation = pyblp.Formulation('1 + prices + sugar + mushy')
product_formulations = (X1_formulation, X2_formulation)
product_formulations
```

```
(prices + Absorb[C(product_ids)], 1 + prices + sugar + mushy)
```

We also need to specify an *Integration* configuration. We consider two alternatives:

1. Monte Carlo draws: we simulate 50 individuals from a random normal distribution. This is just for simplicity. In practice quasi-Monte Carlo sequences such as Halton draws are preferable, and there should generally be many more simulated individuals than just 50.
2. Product rules: we construct nodes and weights according to a product rule that exactly integrates polynomials of degree  $5 \times 2 - 1 = 9$  or less.

```
mc_integration = pyblp.Integration('monte_carlo', size=50, specification_options={'seed': 0})
mc_integration
```

```
Configured to construct nodes and weights with Monte Carlo simulation with options {seed: 0}.
```

```
pr_integration = pyblp.Integration('product', size=5)
pr_integration
```

```
Configured to construct nodes and weights according to the level-5 Gauss-Hermite product rule with options {}.
```

```
mc_problem = pyblp.Problem(product_formulations, product_data, integration=mc_integration)
mc_problem
```



```
Dimensions:
```

```
=====
  T   N   F   I   K1  K2  MD  ED
  --- --- --- --- --- --- ---
94 2256 5 4700 1 4 20 1
=====
```

```
Formulations:
```

```
=====
      Column Indices:      0      1      2      3
  -----
X1: Linear Characteristics  prices
X2: Nonlinear Characteristics  1      prices  sugar  mushy
=====
```

```
pr_problem = pyblp.Problem(product_formulations, product_data, integration=pr_integration)
pr_problem
```

```
Dimensions:
```

```
=====
  T   N   F   I   K1  K2  MD  ED
  --- --- --- --- --- --- ---
94 2256 5 58750 1 4 20 1
=====
```

```
Formulations:
```

```
=====
      Column Indices:      0      1      2      3
  -----
X1: Linear Characteristics  prices
X2: Nonlinear Characteristics  1      prices  sugar  mushy
=====
```

As an illustration of how to configure the optimization routine, we'll use a simpler, non-default *Optimization* configuration that doesn't support parameter bounds, and use a relatively loose tolerance so the problems are solved quickly. In practice along with more integration draws, it's a good idea to use a tighter termination tolerance.

```
bfgs = pyblp.Optimization('bfgs', {'gtol': 1e-4})
bfgs
```

```
Configured to optimize using the BFGS algorithm implemented in SciPy with analytic gradients and options {gtol: +1.0E-
↪04}.
```

We estimate three versions of the model:

1. An unrestricted covariance matrix for random tastes using Monte Carlo integration.
2. An unrestricted covariance matrix for random tastes using the product rule.
3. A restricted diagonal matrix for random tastes using Monte Carlo Integration.

Notice that the only thing that changes when we estimate the restricted covariance is our initial guess of  $\Sigma_0$ . The upper diagonal in this initial guess is ignored because we are optimizing over the lower-triangular Cholesky root of  $\Sigma$ .

```
results1 = mc_problem.solve(sigma=np.ones((4, 4)), optimization=bfgs)
results1
```

Problem Results Summary:

```
=====
GMM   Objective  Gradient      Hessian      Hessian      Clipped      Weighting Matrix  Covariance Matrix
Step  Value       Norm         Min Eigenvalue  Max Eigenvalue  Shares      Condition Number  Condition Number
-----
  2   +1.5E+02   +8.7E-05     +8.5E-02      +6.5E+03       0           +5.2E+07          +8.3E+05
=====
```

Cumulative Statistics:

```
=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
  Time       Converged  Iterations    Evaluations  Iterations  Evaluations
-----
00:03:41    Yes       58            75          87315      267936
=====
```

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):

```
=====
Sigma:      1      prices      sugar      mushy      |      Sigma Squared:      1      prices      sugar      mushy
-----
↪-
  1      +1.2E+00      |      1      +1.5E+00      -1.4E+01      +7.3E-02      -7.1E-01
      (+3.0E+00)      |      (+7.2E+00)      (+5.2E+01)      (+2.2E-01)      (+2.
↪3E+00)
      |
prices     -1.1E+01      +8.4E+00      |      prices     -1.4E+01      +2.0E+02      -1.5E+00      +1.5E+00
      (+1.8E+01)      (+1.2E+01)      |      (+5.2E+01)      (+3.1E+02)      (+1.2E+00)      (+1.
↪5E+01)
      |
=====
```

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```
sugar    +6.1E-02   -9.1E-02   +3.8E-02   |   sugar    +7.3E-02   -1.5E+00   +1.3E-02   +2.0E-02
          (+2.5E-01) (+2.3E-01) (+8.3E-02) |   (+2.2E-01) (+1.2E+00) (+2.8E-02) (+2.7E-
↳01)

mushy    -5.9E-01   -6.2E-01   -2.3E-02   +4.8E-01   |   mushy    -7.1E-01   +1.5E+00   +2.0E-02   +9.6E-01
          (+2.1E+00) (+1.5E+00) (+2.5E+00) (+1.3E+00) |   (+2.3E+00) (+1.5E+01) (+2.7E-01) (+4.
↳0E+00)
```

Beta Estimates (Robust SEs in Parentheses):

```
=====
prices
-----
-3.1E+01
(+6.0E+00)
=====
```

```
results2 = pr_problem.solve(sigma=np.ones((4, 4)), optimization=bfgs)
results2
```

Problem Results Summary:

```
=====
GMM   Objective  Gradient      Hessian      Hessian      Clipped  Weighting Matrix  Covariance Matrix
Step  Value       Norm         Min Eigenvalue Max Eigenvalue Shares   Condition Number  Condition Number
-----
  2   +1.6E+02   +3.2E-05     +1.6E-02     +5.3E+03     0       +5.3E+07          +1.1E+22
=====
```

Cumulative Statistics:

```
=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations Iterations   Evaluations
-----
00:07:25    Yes       64            78         67421       208191
=====
```

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):

```
=====
Sigma:      1      prices      sugar      mushy      |   Sigma Squared:  1      prices      sugar      mushy
-----
↳ -
```

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1	-7.4E-01				1	+5.5E-01	-9.4E+00	+8.3E-02	-1.1E-01
	(+2.1E+00)					(+3.1E+00)	(+3.2E+01)	(+1.7E-01)	(+6.3E-
↪01)									
prices	+1.3E+01	-8.8E-06			prices	-9.4E+00	+1.6E+02	-1.4E+00	+1.9E+00
	(+7.7E+00)	(+2.3E+03)				(+3.2E+01)	(+2.0E+02)	(+7.8E-01)	(+8.
↪8E+00)									
sugar	-1.1E-01	+1.1E-07	-3.0E-09		sugar	+8.3E-02	-1.4E+00	+1.2E-02	-1.7E-02
	(+2.1E-01)	(+1.9E+05)	(+7.6E+02)			(+1.7E-01)	(+7.8E-01)	(+2.1E-02)	(+1.5E-
↪01)									
mushy	+1.5E-01	+4.6E-07	+1.9E-08	-1.3E-08	mushy	-1.1E-01	+1.9E+00	-1.7E-02	+2.2E-02
	(+6.8E-01)	(+3.4E+03)	(+6.3E+02)	(+1.7E+02)		(+6.3E-01)	(+8.8E+00)	(+1.5E-01)	(+2.0E-
↪01)									

Beta Estimates (Robust SEs in Parentheses):

```
=====
prices
-----
-3.1E+01
(+4.4E+00)
=====
```

```
results3 = mc_problem.solve(sigma=np.eye(4), optimization=bfgs)
results3
```

Problem Results Summary:

```
=====
GMM   Objective   Gradient      Hessian      Hessian      Clipped   Weighting Matrix   Covariance Matrix
Step  Value         Norm          Min Eigenvalue  Max Eigenvalue  Shares    Condition Number   Condition Number
-----
  2    +1.8E+02    +1.3E-06     +1.1E+00      +6.0E+03        0         +5.8E+07           +4.8E+04
=====
```

Cumulative Statistics:

```
=====
Computation   Optimizer   Optimization   Objective   Fixed Point   Contraction
Time          Converged   Iterations     Evaluations  Iterations    Evaluations
=====
```

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```

-----
00:01:05      Yes      16      24      18640      57624
=====

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
=====
Sigma:      1      prices      sugar      mushy
-----
1      +5.2E-02
      (+1.1E+00)
prices  +0.0E+00      -4.3E-01
      (+8.0E+00)
sugar   +0.0E+00      +0.0E+00      +3.6E-02
      (+5.8E-02)
mushy   +0.0E+00      +0.0E+00      +0.0E+00      +5.0E-01
      (+1.7E+00)
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-3.0E+01
(+1.4E+00)
=====

```

We see that all three models give similar estimates of the price coefficient  $\hat{\alpha} \approx -30$ . Note a few of the estimated terms on the diagonal of  $\Sigma$  are negative. Since the diagonal consists of standard deviations, negative values are unrealistic. When using another optimization routine that supports bounds (like the default L-BFGS-B routine), these diagonal elements are by default bounded from below by zero.

### Adding Demographics to the Problem

To add demographic data we need to make two changes:

1. We need to load `agent_data`, which for this cereal problem contains pre-computed Monte Carlo draws and demographics.
2. We need to add an `agent_formulation` to the model.

The agent data has several reserved column names.

- `market_ids` are the index that link the agent data to the `market_ids` in product data.
- `weights` are the weights  $w_{it}$  attached to each agent. In each market, these should sum to one so that  $\sum_i w_{it} = 1$ . It is often the case the  $w_{it} = 1/I_t$  where  $I_t$  is the number of agents in market  $t$ , so that each agent gets equal weight. Other possibilities include quadrature nodes and weights.
- `nodes0`, `nodes1`, and so on are the nodes at which the unobserved agent tastes  $\mu_{ijt}$  are evaluated. The nodes should be labeled from  $0, \dots, K_2 - 1$  where  $K_2$  is the number of random coefficients.
- Other fields are the realizations of the demographics  $d$  themselves.

```
agent_data = pd.read_csv(pyblp.data.NEVO_AGENTS_LOCATION)
agent_data.head()
```

	market_ids	city_ids	quarter	weights	nodes0	nodes1	nodes2	\
0	C01Q1		1	1	0.05	0.434101	-1.500838	-1.151079
1	C01Q1		1	1	0.05	-0.726649	0.133182	-0.500750
2	C01Q1		1	1	0.05	-0.623061	-0.138241	0.797441
3	C01Q1		1	1	0.05	-0.041317	1.257136	-0.683054
4	C01Q1		1	1	0.05	-0.466691	0.226968	1.044424

	nodes3	income	income_squared	age	child
0	0.161017	0.495123	8.331304	-0.230109	-0.230851
1	0.129732	0.378762	6.121865	-2.532694	0.769149
2	-0.795549	0.105015	1.030803	-0.006965	-0.230851
3	0.259044	-1.485481	-25.583605	-0.827946	0.769149
4	0.092019	-0.316597	-6.517009	-0.230109	-0.230851

The agent formulation tells us which columns of demographic information to interact with  $X_2$ .

```
agent_formulation = pyblp.Formulation('0 + income + income_squared + age + child')
agent_formulation
```

```
income + income_squared + age + child
```

This tells us to include demographic realizations for `income`, `income_squared`, `age`, and the presence of children, `child`, but to ignore other possible demographics when interacting demographics with  $X_2$ . We should also generally exclude the constant from the demographic formula.

Now we configure the *Problem* to include the `agent_formulation` and `agent_data`, which follow the `product_formulations` and `product_data`.

When we display the class, it lists the demographic interactions in the table of formulations and reports  $D = 4$ , the dimension of the demographic draws.

```

nevo_problem = pyblp.Problem(
    product_formulations,
    product_data,
    agent_formulation,
    agent_data
)
nevo_problem

```

Dimensions:

```

=====
T      N      F      I      K1     K2     D      MD     ED
---    ---    ---    ---    ---    ---    ---    ---    ---
94    2256    5     1880    1      4      4      20     1
=====

```

Formulations:

```

=====
Column Indices:      0      1      2      3
-----
X1: Linear Characteristics  prices
X2: Nonlinear Characteristics  1      prices  sugar  mushy
d: Demographics  income  income_squared  age  child
=====

```

The initialized problem can be solved with *Problem.solve*. We'll use the same starting values as *Nevo (2000)*. By passing a diagonal matrix as starting values for  $\Sigma$ , we're choosing to ignore covariance terms. Similarly, zeros in the starting values for  $\Pi$  mean that those parameters will be fixed at zero.

To replicate common estimates, we'll use the non-default BFGS optimization routine (with a slightly tighter tolerance to avoid getting stuck at a spurious local minimum), and we'll configure *Problem.solve* to use 1-step GMM instead of 2-step GMM.

```

initial_sigma = np.diag([0.3302, 2.4526, 0.0163, 0.2441])
initial_pi = np.array([
    [ 5.4819,  0,      0.2037,  0      ],
    [15.8935, -1.2000,  0,      2.6342],
    [-0.2506,  0,      0.0511,  0      ],
    [ 1.2650,  0,     -0.8091,  0      ]
])
tighter_bfgs = pyblp.Optimization('bfgs', {'gtol': 1e-5})
nevo_results = nevo_problem.solve(
    initial_sigma,
    initial_pi,
    optimization=tighter_bfgs,

```

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```

method='1s'
)
nevo_results
Problem Results Summary:
=====
GMM   Objective   Gradient      Hessian      Hessian      Clipped   Weighting Matrix  Covariance Matrix
Step  Value        Norm          Min Eigenvalue Max Eigenvalue Shares   Condition Number  Condition Number
-----
1     +4.6E+00    +6.9E-06     +2.7E-05     +1.6E+04     0       +6.9E+07         +8.4E+08
=====

Cumulative Statistics:
=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations Iterations   Evaluations
-----
00:02:11    Yes       51            57         45483       141195
=====

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
=====
Sigma:      1          prices      sugar       mushy       |  Pi:      income     income_squared  age         child
-----
1          +5.6E-01   (+1.6E-01)   |  1         +2.3E+00   (+1.2E+00)   |  +1.3E+00   (+6.3E-01)   +0.0E+00
prices     +0.0E+00   +3.3E+00   (+1.3E+00)   |  prices    +5.9E+02   -3.0E+01   (+1.4E+01)   +0.0E+00   +1.1E+01   (+4.1E+00)
sugar      +0.0E+00   +0.0E+00   -5.8E-03   (+1.4E-02)   |  sugar     -3.8E-01   +0.0E+00   (+1.2E-01)   +5.2E-02   (+2.6E-02)   +0.0E+00
mushy      +0.0E+00   +0.0E+00   +0.0E+00   +9.3E-02   (+1.9E-01)   |  mushy     +7.5E-01   +0.0E+00   (+8.0E-01)   -1.4E+00   (+6.7E-01)   +0.0E+00
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----

```

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```
-6.3E+01
(+1.5E+01)
=====
```

Results are similar to those in the original paper with a (scaled) objective value of  $q(\hat{\theta}) = 4.65$  and a price coefficient of  $\hat{\alpha} = -62.7$ .

### Restricting Parameters

Because the interactions between `price`, `income`, and `income_squared` are potentially collinear, we might worry that  $\hat{\Pi}_{21} = 588$  and  $\hat{\Pi}_{22} = -30.2$  are pushing the price coefficient in opposite directions. Both are large in magnitude but statistically insignificant. One way of dealing with this is to restrict  $\Pi_{22} = 0$ .

There are two ways we can do this:

1. Change the initial  $\Pi_0$  values to make this term zero.
2. Change the agent formula to drop `income_squared`.

First, we'll change the initial  $\Pi_0$  values.

```
restricted_pi = initial_pi.copy()
restricted_pi[1, 1] = 0
nevo_problem.solve(
    initial_sigma,
    restricted_pi,
    optimization=tighter_bfgs,
    method='ls'
)
```

Problem Results Summary:

```
=====
GMM   Objective  Gradient      Hessian      Hessian      Clipped  Weighting Matrix  Covariance Matrix
Step  Value       Norm         Min Eigenvalue  Max Eigenvalue  Shares   Condition Number  Condition Number
-----
  1   +1.5E+01   +5.2E-06     +4.7E-02      +1.7E+04        0        +6.9E+07          +5.7E+05
=====
```

Cumulative Statistics:

```
=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations  Iterations   Evaluations
-----
```

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```

00:01:22      Yes      34      40      33651      104191
=====
Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
=====
Sigma:      1      prices      sugar      mushy      |      Pi:      income      income_squared      age      child
-----
1      +3.7E-01      |      1      +3.1E+00      +0.0E+00      +1.2E+00      +0.0E+00
      (+1.2E-01)      |      |      (+1.1E+00)      |      (+1.0E+00)
prices      +0.0E+00      +1.8E+00      |      prices      +4.2E+00      +0.0E+00      +0.0E+00      +1.2E+01
      (+9.2E-01)      |      |      (+4.6E+00)      |      (+5.2E+00)
sugar      +0.0E+00      +0.0E+00      -4.4E-03      |      sugar      -1.9E-01      +0.0E+00      +2.8E-02      +0.0E+00
      (+1.2E-02)      |      |      (+3.5E-02)      |      (+3.2E-02)
mushy      +0.0E+00      +0.0E+00      +0.0E+00      +8.6E-02      |      mushy      +1.5E+00      +0.0E+00      -1.5E+00      +0.0E+00
      (+1.9E-01)      |      |      (+6.5E-01)      |      (+1.1E+00)
=====
Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-3.2E+01
(+2.3E+00)
=====

```

Now we'll drop both `income_squared` and the corresponding column in  $\Pi_0$ .

```

restricted_formulation = pyblp.Formulation('0 + income + age + child')
deleted_pi = np.delete(initial_pi, 1, axis=1)
restricted_problem = pyblp.Problem(
    product_formulations,
    product_data,
    restricted_formulation,
    agent_data
)
restricted_problem.solve(
    initial_sigma,

```

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```

deleted_pi,
optimization=tighter_bfgs,
method='ls'
)

```

Problem Results Summary:

```

=====
GMM   Objective  Gradient      Hessian      Hessian      Clipped  Weighting Matrix  Covariance Matrix
Step  Value       Norm         Min Eigenvalue Max Eigenvalue Shares   Condition Number  Condition Number
-----
1     +1.5E+01   +5.2E-06    +4.7E-02     +1.7E+04     0       +6.9E+07          +5.7E+05
=====

```

Cumulative Statistics:

```

=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations Iterations   Evaluations
-----
00:01:22    Yes       34            40         33639       104205
=====

```

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):

```

=====
Sigma:      1          prices      sugar      mushy      |  Pi:      income      age      child
-----
1          +3.7E-01      (+1.2E-01)      |  1          +3.1E+00      +1.2E+00      +0.0E+00
                                         |           (+1.1E+00)      (+1.0E+00)
prices     +0.0E+00      +1.8E+00      (+9.2E-01)      |  prices     +4.2E+00      +0.0E+00      +1.2E+01
                                         |           (+4.6E+00)      (+5.2E+00)
sugar      +0.0E+00      +0.0E+00      -4.4E-03      (+1.2E-02)      |  sugar      -1.9E-01      +2.8E-02      +0.0E+00
                                         |           (+3.5E-02)      (+3.2E-02)
mushy      +0.0E+00      +0.0E+00      +0.0E+00      +8.6E-02      (+1.9E-01)      |  mushy      +1.5E+00      -1.5E+00      +0.0E+00
                                         |           (+6.5E-01)      (+1.1E+00)
=====

```

Beta Estimates (Robust SEs in Parentheses):

```

=====
prices

```

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```
-----  
-3.2E+01  
(+2.3E+00)  
=====
```

The parameter estimates and standard errors are identical for both approaches. Based on the number of fixed point iterations, there is some evidence that the solver took a slightly different path for each problem, but both restricted problems arrived at identical answers. The  $\hat{\Pi}_{12}$  interaction term is still insignificant.

---

The [online version](#) of the following section may be easier to read.

---

## 4.3 Random Coefficients Logit Tutorial with the Automobile Data

```
import pyblp
import numpy as np
import pandas as pd

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

In this tutorial, we'll use data from *Berry, Levinsohn, and Pakes (1995)* to solve the paper's automobile problem.

### 4.3.1 Application of Random Coefficients Logit with the Automobile Data

This tutorial is similar to the *fake cereal tutorial*, but exhibits some other features of pyblp:

- Incorporating a supply side into demand estimation.
- Allowing for simple price-income demographic effects.
- Calculating clustered standard errors.

#### Loading the Data

We'll use `pandas` to load two sets of data:

1. `product_data`, which contains prices, shares, and other product characteristics.
2. `agent_data`, which contains draws from the distribution of heterogeneity.

```
product_data = pd.read_csv(pyblp.data.BLP_PRODUCTS_LOCATION)
product_data.head()
```

```

market_ids clustering_ids car_ids firm_ids region shares prices \
0 1971 AMGREM71 129 15 US 0.001051 4.935802
1 1971 AMHORN71 130 15 US 0.000670 5.516049
2 1971 AMJAVL71 132 15 US 0.000341 7.108642
3 1971 AMMATA71 134 15 US 0.000522 6.839506
4 1971 AMAMBS71 136 15 US 0.000442 8.928395

hpwt air mpd ... supply_instruments2 supply_instruments3 \
0 0.528997 0 1.888146 ... 0.0 1.705933
1 0.494324 0 1.935989 ... 0.0 1.680910
2 0.467613 0 1.716799 ... 0.0 1.801067
3 0.426540 0 1.687871 ... 0.0 1.818061
4 0.452489 0 1.504286 ... 0.0 1.933210

supply_instruments4 supply_instruments5 supply_instruments6 \
0 1.595656 87.0 -61.959985
1 1.490295 87.0 -61.959985
2 1.357703 87.0 -61.959985
3 1.261347 87.0 -61.959985
4 1.237365 87.0 -61.959985

supply_instruments7 supply_instruments8 supply_instruments9 \
0 0.0 46.060389 29.786989
1 0.0 46.060389 29.786989
2 0.0 46.060389 29.786989
3 0.0 46.060389 29.786989
4 0.0 46.060389 29.786989

supply_instruments10 supply_instruments11
0 0.0 1.888146
1 0.0 1.935989
2 0.0 1.716799
3 0.0 1.687871
4 0.0 1.504286

[5 rows x 33 columns]

```

The `product_data` contains market IDs, product IDs, firm IDs, shares, prices, a number of product characteristics, and instruments. The product IDs are called `clustering_ids` because they will be used to compute clustered standard errors. For more information about the instruments and the example data as a whole, refer to the `data` module.

The `agent_data` contains market IDs, integration weights  $w_{it}$ , integration nodes  $\nu_{it}$ , and demographics  $d_{it}$ . Here we use the  $I_t = 200$  importance sampling

weights and nodes from the original paper.

In non-example problems, it is usually a better idea to use many more draws, or a more sophisticated *Integration* configuration such as sparse grid quadrature.

```
agent_data = pd.read_csv(pyblp.data.BLP_AGENTS_LOCATION)
agent_data.head()

   market_ids  weights  nodes0  nodes1  nodes2  nodes3  nodes4  \
0         1971  0.000543  1.192188  0.478777  0.980830 -0.824410  2.473301
1         1971  0.000723  1.497074 -2.026204 -1.741316  1.412568 -0.747468
2         1971  0.000544  1.438081  0.813280 -1.749974 -1.203509  0.049558
3         1971  0.000701  1.768655 -0.177453  0.286602  0.391517  0.683669
4         1971  0.000549  0.849970 -0.135337  0.735920  1.036247 -1.143436

   income
0  109.560369
1   45.457314
2  127.146548
3   22.604045
4  170.226032
```

### Setting up the Problem

Unlike the fake cereal problem, we won't absorb any fixed effects in the automobile problem, so the linear part of demand  $X_1$  has more components. We also need to specify a formula for the random coefficients  $X_2$ , including a random coefficient on the constant, which captures correlation among all inside goods.

The primary new addition to the model relative to the fake cereal problem is that we add a supply side formula for product characteristics that contribute to marginal costs,  $X_3$ . The *patsy*-style formulas support functions of regressors such as the `log` function used below.

We stack the three product formulations in order:  $X_1$ ,  $X_2$ , and  $X_3$ .

```
product_formulations = (
    pyblp.Formulation('1 + hpwt + air + mpd + space'),
    pyblp.Formulation('1 + prices + hpwt + air + mpd + space'),
    pyblp.Formulation('1 + log(hpwt) + air + log(mpg) + log(space) + trend')
)
product_formulations

(1 + hpwt + air + mpd + space,
 1 + prices + hpwt + air + mpd + space,
 1 + log(hpwt) + air + log(mpg) + log(space) + trend)
```

The original specification for the automobile problem includes the term  $\log(y_i - p_j)$ , in which  $y$  is income and  $p$  are prices. Instead of including this term, which gives rise to a host of numerical problems, we'll follow *Berry, Levinsohn, and Pakes (1999)* and use its first-order linear approximation,  $p_j/y_i$ .

The agent formulation for demographics,  $d$ , includes a column of  $1/y_i$  values, which we'll interact with  $p_j$ . To do this, we will treat draws of  $y_i$  as demographic variables.

```
agent_formulation = pyblp.Formulation('0 + I(1 / income)')
agent_formulation
I(1 / income)
```

As in the cereal example, the *Problem* can be constructed by combining the `product_formulations`, `product_data`, `agent_formulation`, and `agent_data`. We'll also choose the functional form of marginal costs  $c_{jt}$ . A linear marginal cost specification is the default setting, so we'll need to use the `costs_type` argument of *Problem* to employ the log-linear specification used by *Berry, Levinsohn, and Pakes (1995)*.

```
problem = pyblp.Problem(product_formulations, product_data, agent_formulation, agent_data, costs_type='log')
problem
```

```
Dimensions:
=====
 T      N      F      I      K1      K2      K3      D      MD      MS
-----
 20    2217    26    4000     5       6       6       1     13     18
=====

Formulations:
=====
      Column Indices:      0      1      2      3      4      5
-----
X1: Linear Characteristics      1      hpwt      air      mpd      space
X2: Nonlinear Characteristics      1      prices      hpwt      air      mpd      space
X3: Log Cost Characteristics      1      log(hpwt)      air      log(mpg)      log(space)      trend
      d: Demographics      1/income
=====
```

The problem outputs a table of dimensions:

- $T$  denotes the number of markets.
- $N$  is the length of the dataset (the number of products across all markets).
- $F$  denotes the number of firms.
- $I = \sum_t I_t$  is the total number of agents across all markets (200 draws per market times 20 markets).



- $K_1$  is the number of linear demand characteristics.
- $K_2$  is the number of nonlinear demand characteristics.
- $K_3$  is the number of linear supply characteristics.
- $D$  is the number of demographic variables.
- $M_D$  is the number of demand instruments, including exogenous regressors.
- $M_S$  is the number of supply instruments, including exogenous regressors.

The formulations table describes all four formulas for demand-side linear characteristics, demand-side nonlinear characteristics, supply-side characteristics, and demographics.

### Solving the Problem

The only remaining decisions are:

- Choosing  $\Sigma$  and  $\Pi$  starting values,  $\Sigma_0$  and  $\Pi_0$ .
- Potentially choosing bounds for  $\Sigma$  and  $\Pi$ .

The decisions we will use are:

- Use published estimates as our starting values in  $\Sigma_0$ .
- Interact the inverse of income,  $1/y_i$ , only with prices, and use the published estimate on  $\log(y_i - p_j)$  as our starting value for  $\alpha$  in  $\Pi_0$ .
- Bound  $\Sigma_0$  to be positive since it is a diagonal matrix where the diagonal consists of standard deviations.

When using a routine that supports bounds, it's usually a good idea to set your own more bounds so that the routine doesn't try out large parameter values that create numerical issues.

```
initial_sigma = np.diag([3.612, 0, 4.628, 1.818, 1.050, 2.056])
initial_pi = np.c_[0, -43.501, 0, 0, 0, 0]
```

Note that there are only 5 nonzeros on the diagonal of  $\Sigma$ , which means that we only need 5 columns of integration nodes to integrate over these 5 dimensions of unobserved heterogeneity. Indeed, `agent_data` contains exactly 5 columns of nodes. If we were to ignore the  $\log(y_i - p_j)$  term (by not configuring  $\Pi$ ) and include a term on prices in  $\Sigma$  instead, we would have needed 6 columns of integration nodes in our `agent_data`.

A downside of the log-linear marginal costs specification is that nonpositive estimated marginal costs can create problems for the optimization routine when computing  $\log c(\hat{\theta})$ . We'll use the `costs_bounds` argument to bound marginal costs from below by a small number.

Finally, as in the original paper, we'll use `W_type` and `se_type` to cluster by product IDs, which were specified as `clustering_ids` in `product_data`, and set `initial_update=True` to update the initial GMM weighting matrix and the mean utility at the starting parameter values.

```

results = problem.solve(
    initial_sigma,
    initial_pi,
    costs_bounds=(0.001, None),
    W_type='clustered',
    se_type='clustered',
    initial_update=True,
)
results

```

Problem Results Summary:

```

=====
GMM   Objective   Projected   Reduced Hessian  Reduced Hessian  Clipped  Clipped  Weighting Matrix  Covariance Matrix
Step  Value        Gradient Norm Min Eigenvalue   Max Eigenvalue   Shares   Costs   Condition Number  Condition Number
-----
  2   +5.0E+02    +3.6E-08    +4.9E-01        +5.1E+02        0        0        +4.2E+09          +3.8E+08
=====

```

Cumulative Statistics:

```

=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations Iterations   Evaluations
-----
00:28:13    Yes       58            94         28678       87850
=====

```

Nonlinear Coefficient Estimates (Robust SEs Adjusted for 999 Clusters in Parentheses):

```

=====
Sigma:      1      prices      hpwt      air      mpd      space      |      Pi:      1/income
-----
  1      +2.0E+00
      (+6.1E+00)
prices    +0.0E+00  +0.0E+00
hpwt      +0.0E+00  +0.0E+00  +6.1E+00
      (+2.2E+00)
air       +0.0E+00  +0.0E+00  +0.0E+00  +4.0E+00
      (+2.1E+00)
-----
|      1      +0.0E+00
|      prices  -4.5E+01
|              (+9.2E+00)
|      hpwt   +0.0E+00
|      air    +0.0E+00
|
=====

```

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```

mpd      +0.0E+00  +0.0E+00  +0.0E+00  +0.0E+00  +2.5E-01
                                         (+5.5E-01)
space    +0.0E+00  +0.0E+00  +0.0E+00  +0.0E+00  +0.0E+00  +1.9E+00
                                         (+1.1E+00)
=====
Beta Estimates (Robust SEs Adjusted for 999 Clusters in Parentheses):
=====
      1          hpwt          air          mpd          space
-----
-7.3E+00  +3.5E+00  -1.0E+00  +4.2E-01  +4.2E+00
(+2.8E+00) (+1.4E+00) (+2.1E+00) (+2.5E-01) (+6.6E-01)
=====
Gamma Estimates (Robust SEs Adjusted for 999 Clusters in Parentheses):
=====
      1          log (hpwt)          air          log (mpg)          log (space)          trend
-----
+2.8E+00  +9.0E-01  +4.2E-01  -5.2E-01  -2.6E-01  +2.7E-02
(+1.2E-01) (+7.2E-02) (+8.7E-02) (+7.3E-02) (+2.1E-01) (+3.1E-03)
=====

```

There are some discrepancies between our results and the original paper, but results are similar.

---

The [online version](#) of the following section may be easier to read.

---

## 4.4 Post-Estimation Tutorial

```
%matplotlib inline

import pyblp
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

This tutorial covers several features of `pyblp` which are available after estimation including:

1. Calculating elasticities and diversion ratios.
2. Calculating marginal costs and markups.
3. Computing the effects of mergers: prices, shares, and HHI.
4. Using a parametric bootstrap to estimate standard errors.
5. Estimating optimal instruments.

### 4.4.1 Problem Results

As in the *fake cereal tutorial*, we'll first solve the fake cereal problem from *Nevo (2000)*. We load the fake data and estimate the model as in the previous tutorial. We output the setup of the model to confirm we have correctly configured the *Problem*

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
agent_data = pd.read_csv(pyblp.data.NEVO_AGENTS_LOCATION)
product_formulations = (
    pyblp.Formulation('0 + prices', absorb='C(product_ids)'),
```

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```

pyblp.Formulation('1 + prices + sugar + mushy')
)
agent_formulation = pyblp.Formulation('0 + income + income_squared + age + child')
problem = pyblp.Problem(product_formulations, product_data, agent_formulation, agent_data)
problem

```

Dimensions:

```

=====
T      N      F      I      K1     K2     D      MD     ED
---    ---    ---    ---    ---    ---    ---    ---    ---
94    2256    5     1880    1      4      4      20     1
=====

```

Formulations:

```

=====
Column Indices:      0          1          2          3
-----
X1: Linear Characteristics  prices
X2: Nonlinear Characteristics  1          prices      sugar      mushy
   d: Demographics          income  income_squared  age      child
=====

```

We'll solve the problem in the same way as before. The `Problem.solve` method returns a `ProblemResults` class, which displays basic estimation results. The results that are displayed are simply formatted information extracted from various class attributes such as `ProblemResults.sigma` and `ProblemResults.sigma_se`.

```

initial_sigma = np.diag([0.3302, 2.4526, 0.0163, 0.2441])
initial_pi = [
    [ 5.4819,  0,      0.2037,  0      ],
    [15.8935, -1.2000,  0,      2.6342],
    [-0.2506,  0,      0.0511,  0      ],
    [ 1.2650,  0,      -0.8091,  0      ]
]
results = problem.solve(
    initial_sigma,
    initial_pi,
    optimization=pyblp.Optimization('bfgs', {'gtol': 1e-5}),
    method='ls'
)
results

```

Problem Results Summary:

(continues on next page)

(continued from previous page)

```

=====
GMM      Objective   Gradient      Hessian        Hessian        Clipped   Weighting Matrix   Covariance Matrix
Step     Value          Norm          Min Eigenvalue  Max Eigenvalue  Shares    Condition Number   Condition Number
-----
  1      +4.6E+00      +6.9E-06      +2.7E-05        +1.6E+04         0          +6.9E+07           +8.4E+08
=====

Cumulative Statistics:
=====
Computation Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged     Iterations    Evaluations Iterations   Evaluations
-----
00:02:08      Yes           51            57          45483       141195
=====

Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
=====
Sigma:      1          prices      sugar      mushy      |  Pi:      income      income_squared      age      child
-----
  1          +5.6E-01      (+1.6E-01)      |  1          +2.3E+00      +0.0E+00      +1.3E+00      +0.0E+00
                                     |          (+1.2E+00)                                     (+6.3E-01)
prices      +0.0E+00      +3.3E+00      |  prices      +5.9E+02      -3.0E+01      +0.0E+00      +1.1E+01
                                     |          (+2.7E+02)      (+1.4E+01)                                     (+4.1E+00)
                                     |          (+1.3E+00)
sugar       +0.0E+00      +0.0E+00      -5.8E-03      |  sugar       -3.8E-01      +0.0E+00      +5.2E-02      +0.0E+00
                                     |          (+1.4E-02)                                     (+2.6E-02)
mushy       +0.0E+00      +0.0E+00      +0.0E+00      +9.3E-02      |  mushy       +7.5E-01      +0.0E+00      -1.4E+00      +0.0E+00
                                     |          (+1.9E-01)                                     (+6.7E-01)
=====

Beta Estimates (Robust SEs in Parentheses):
=====
prices
-----
-6.3E+01
(+1.5E+01)
=====

```

Additional post-estimation outputs can be computed with *ProblemResults* methods.

### 4.4.2 Elasticities and Diversion Ratios

We can estimate elasticities,  $\varepsilon$ , and diversion ratios,  $\mathcal{D}$ , with `ProblemResults.compute_elasticities` and `ProblemResults.compute_diversion_ratios`.

As a reminder, elasticities in each market are

$$\varepsilon_{jk} = \frac{x_k}{s_j} \frac{\partial s_j}{\partial x_k}. \quad (4.16)$$

Diversion ratios are

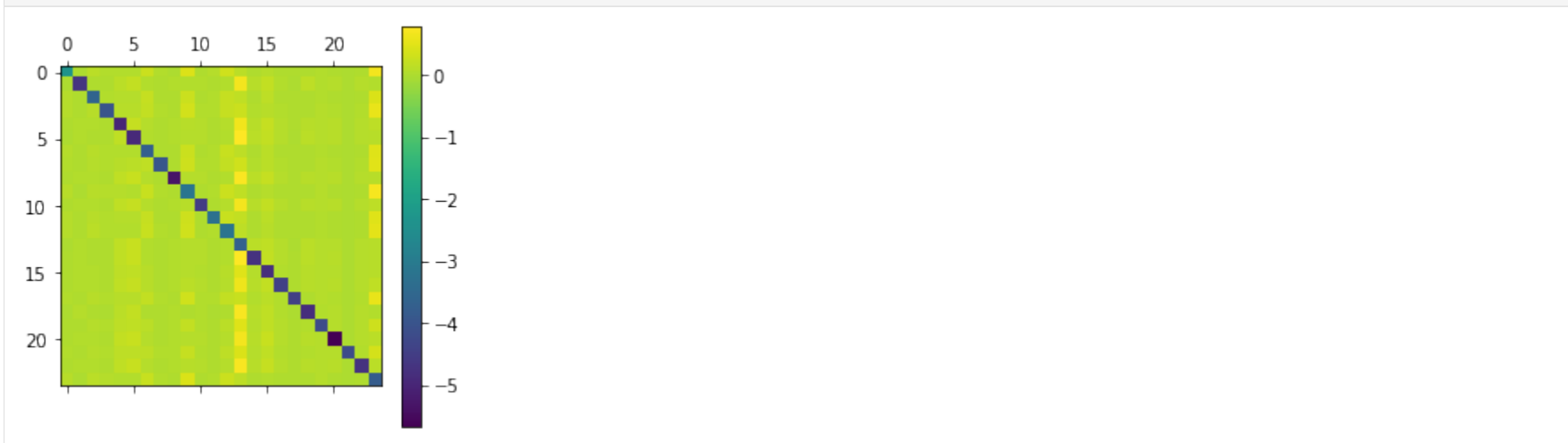
$$\mathcal{D}_{jk} = -\frac{\partial s_k}{\partial x_j} / \frac{\partial s_j}{\partial x_j}. \quad (4.17)$$

Following *Conlon and Mortimer (2018)*, we report the diversion to the outside good  $D_{j0}$  on the diagonal instead of  $D_{jj} = -1$ .

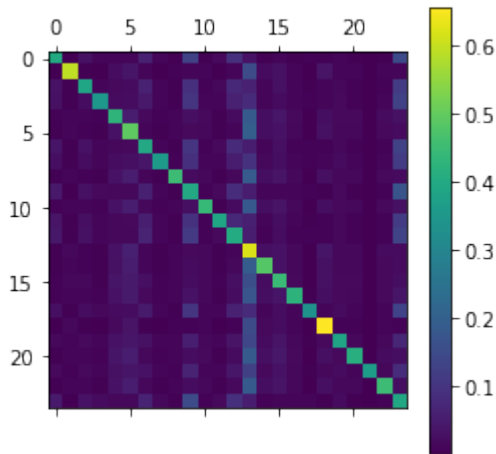
```
elasticities = results.compute_elasticities()
diversions = results.compute_diversion_ratios()
```

Post-estimation outputs are computed for each market and stacked. We'll use `matplotlib` functions to display the matrices associated with a single market.

```
single_market = product_data['market_ids'] == 'C01Q1'
plt.colorbar(plt.matshow(elasticities[single_market]));
```



```
plt.colorbar(plt.matshow(diversions[single_market]));
```



The diagonal of the first image consists of own elasticities and the diagonal of the second image consists of diversion ratios to the outside good. As one might expect, own price elasticities are large and negative while cross-price elasticities are positive but much smaller.

Elasticities and diversion ratios can be computed with respect to variables other than prices with the name argument of *ProblemResults.compute\_elasticities* and *ProblemResults.compute\_diversion\_ratios*. Additionally, *ProblemResults.compute\_long\_run\_diversion\_ratios* can be used to understand substitution when products are eliminated from the choice set.

The convenience methods *ProblemResults.extract\_diagonals* and *ProblemResults.extract\_diagonal\_means* can be used to extract information about own elasticities of demand from elasticity matrices.

```
means = results.extract_diagonal_means(elasticities)
```

An alternative to summarizing full elasticity matrices is to use *ProblemResults.compute\_aggregate\_elasticities* to estimate aggregate elasticities of demand,  $E$ , in each market, which reflect the change in total sales under a proportional sales tax of some factor.

```
aggregates = results.compute_aggregate_elasticities(factor=0.1)
```

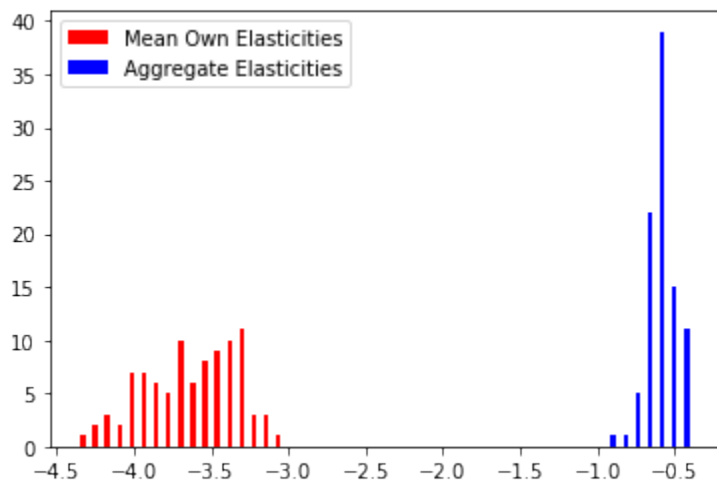
Since demand for an entire product category is generally less elastic than the average elasticity of individual products, mean own elasticities are generally larger in magnitude than aggregate elasticities.

```
plt.hist(
    [means.flatten(), aggregates.flatten()],
```

(continues on next page)



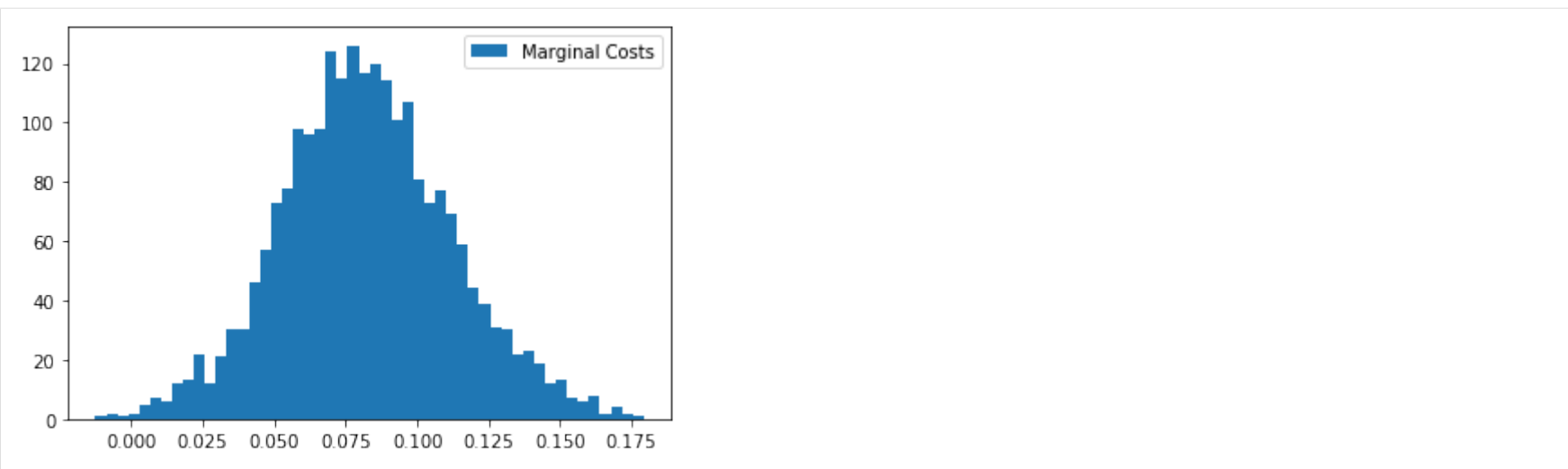
```
color=['red', 'blue'],
bins=50
);
plt.legend(['Mean Own Elasticities', 'Aggregate Elasticities']);
```



### 4.4.3 Marginal Costs and Markups

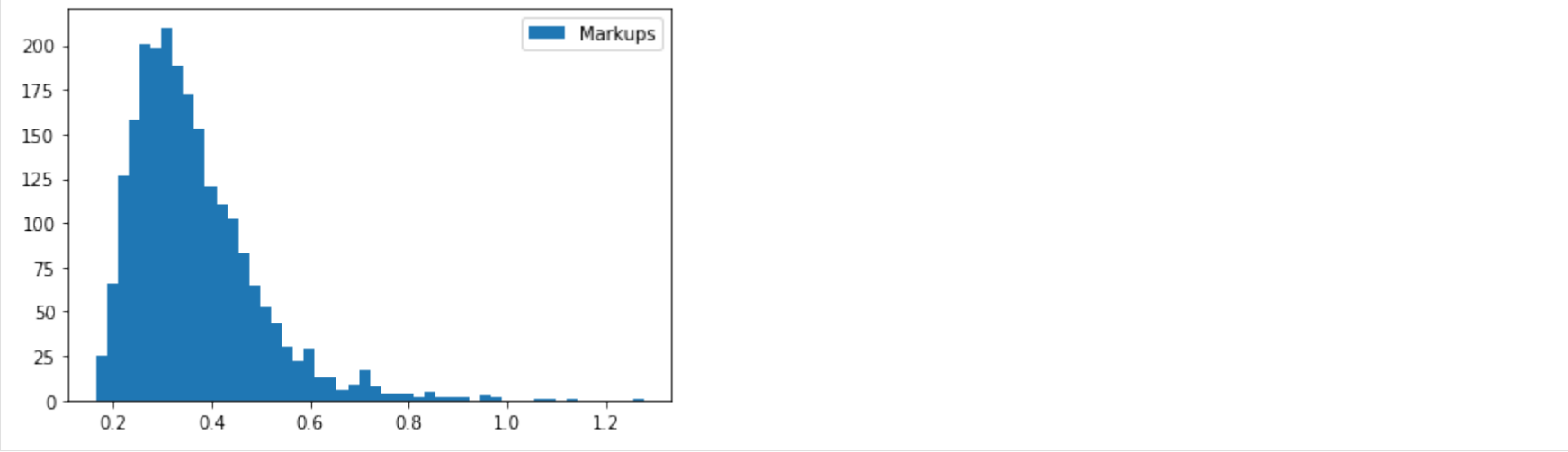
To compute marginal costs,  $c$ , the `product_data` passed to `Problem` must have had a `firm_ids` field. Since we included firm IDs when configuring the problem, we can use `ProblemResults.compute_costs`.

```
costs = results.compute_costs()
plt.hist(costs, bins=50);
plt.legend(["Marginal Costs"]);
```



Other methods that compute supply-side outputs often compute marginal costs themselves. For example, `ProblemResults.compute_markup`s will compute marginal costs when estimating markups,  $\mu$ , but computation can be sped up if we just use our pre-computed values.

```
markups = results.compute_markup(costs=costs)
plt.hist(markups, bins=50);
plt.legend(["Markups"]);
```



#### 4.4.4 Mergers

Before computing post-merger outputs, we'll supplement our pre-merger markups with some other outputs. We'll compute Herfindahl-Hirschman Indices, HHI, with `ProblemResults.compute_hhi`; population-normalized gross expected profits,  $\pi$ , with `ProblemResults.compute_profits`; and population-normalized consumer surpluses, CS, with `ProblemResults.compute_consumer_surpluses`.

```
hhi = results.compute_hhi()
profits = results.compute_profits(costs=costs)
cs = results.compute_consumer_surpluses()
```

To compute post-merger outputs, we'll create a new set of firm IDs that represent a merger of firms 2 and 1.

```
product_data['merger_ids'] = product_data['firm_ids'].replace(2, 1)
```

We can use `ProblemResults.compute_approximate_prices` or `ProblemResults.compute_prices` to estimate post-merger prices. The first method, which is discussed, for example, in *Nevo (1997)*, assumes that shares and their price derivatives are unaffected by the merger. The second method does not make these assumptions and iterates over the  $\zeta$ -markup equation from *Morrow and Skerlos (2011)* to solve the full system of  $J_t$  equations and  $J_t$  unknowns in each market  $t$ . We'll use the latter, since it is fast enough for this example problem.

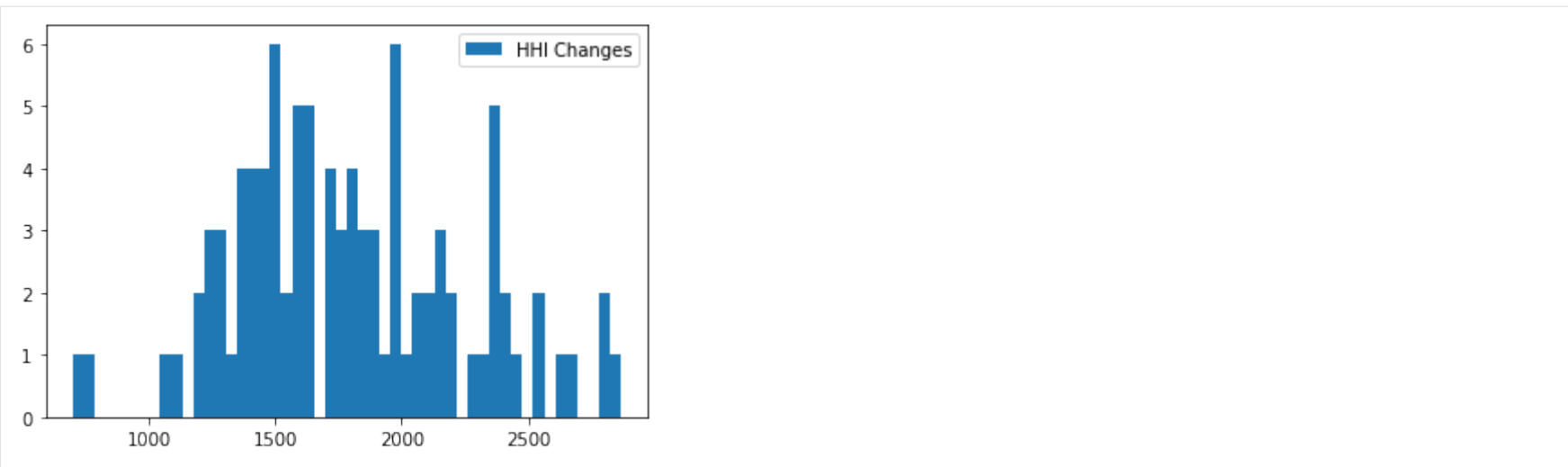
```
changed_prices = results.compute_prices(
    firm_ids=product_data['merger_ids'],
    costs=costs
)
```

We'll compute post-merger shares with `ProblemResults.compute_shares`.

```
changed_shares = results.compute_shares(changed_prices)
```

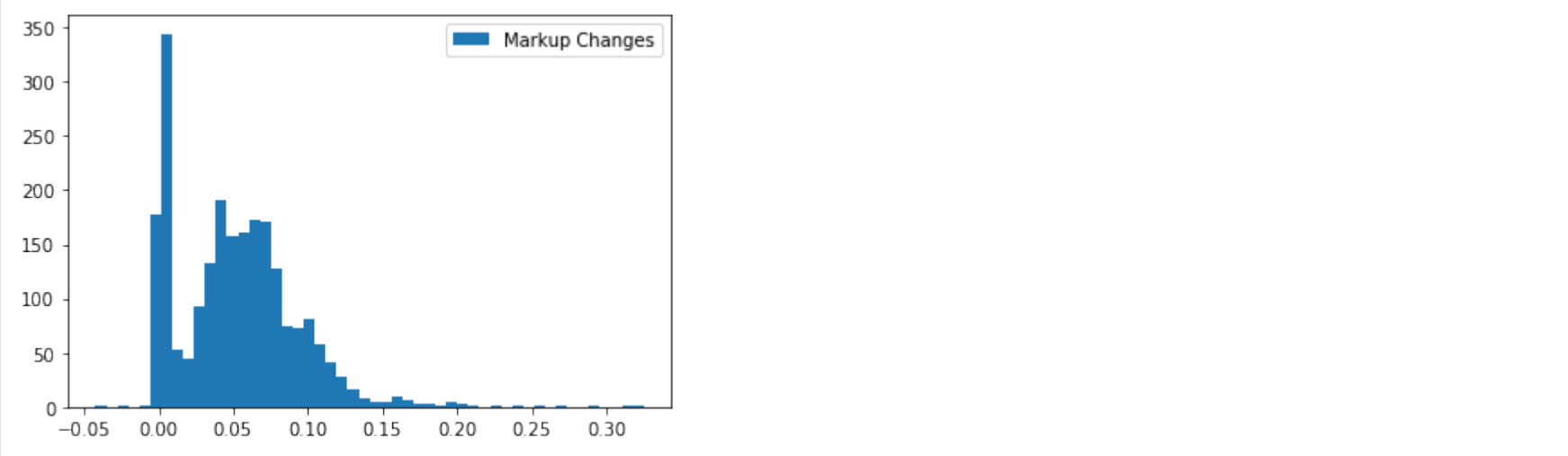
Post-merger prices and shares are used to compute other post-merger outputs. For example, HHI increases.

```
changed_hhi = results.compute_hhi(
    firm_ids=product_data['merger_ids'],
    shares=changed_shares
)
plt.hist(changed_hhi - hhi, bins=50);
plt.legend(["HHI Changes"]);
```

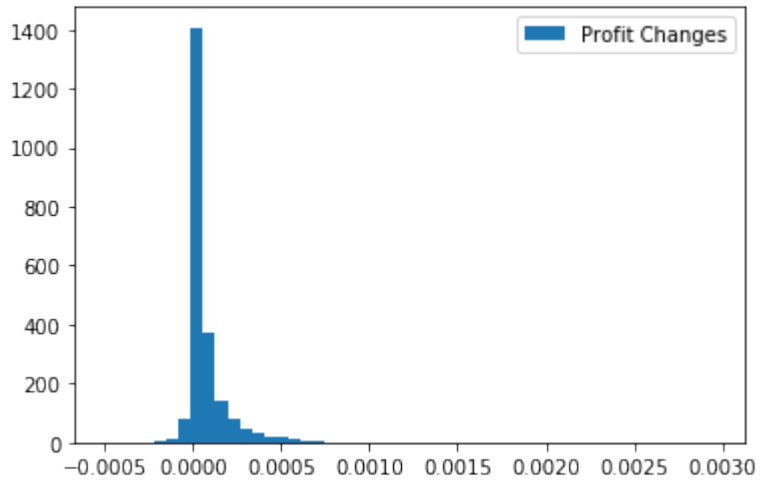


Markups,  $\mathcal{M}$ , and profits,  $\pi$ , generally increase as well.

```
changed_markups = results.compute_markups(changed_prices, costs)
plt.hist(changed_markups - markups, bins=50);
plt.legend(["Markup Changes"]);
```

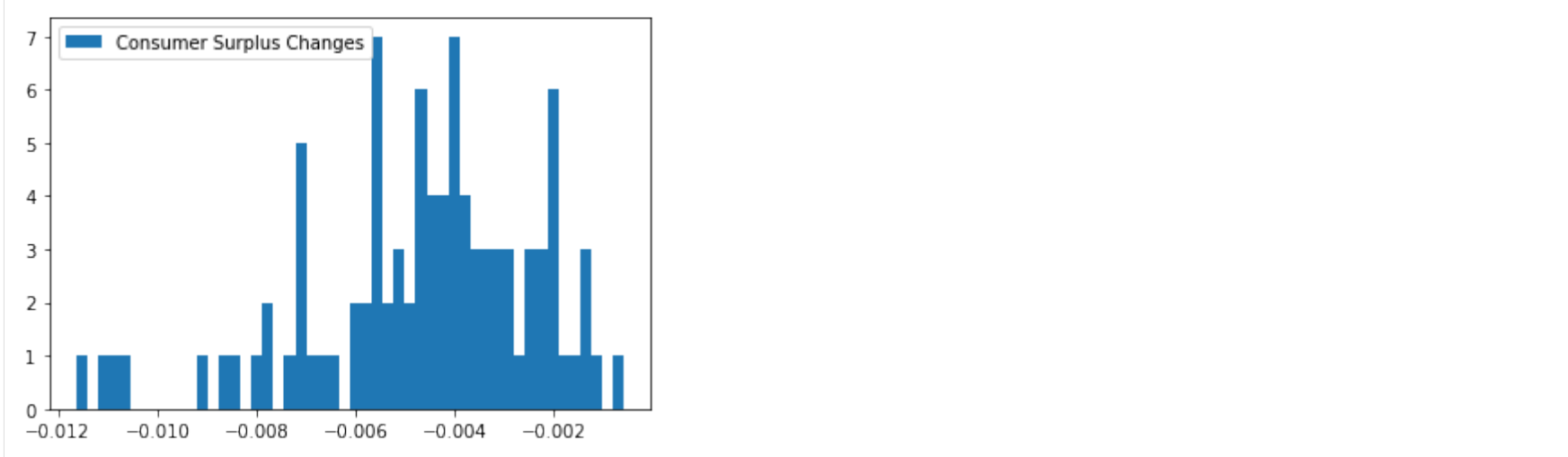


```
changed_profits = results.compute_profits(changed_prices, changed_shares, costs)
plt.hist(changed_profits - profits, bins=50);
plt.legend(["Profit Changes"]);
```



On the other hand, consumer surpluses, CS, generally decrease.

```
changed_cs = results.compute_consumer_surpluses(changed_prices)
plt.hist(changed_cs - cs, bins=50);
plt.legend(["Consumer Surplus Changes"]);
```



#### 4.4.5 Bootstrapping Results

Post-estimation outputs can be informative, but they don't mean much without a sense sample-to-sample variability. One way to estimate confidence intervals for post-estimation outputs is with a standard bootstrap procedure:

1. Construct a large number of bootstrap samples by sampling with replacement from the original product data.
2. Initialize and solve a *Problem* for each bootstrap sample.
3. Compute the desired post-estimation output for each bootstrapped *ProblemResults* and from the resulting empirical distribution, construct bootstrap confidence intervals.

Although appealing because of its simplicity, the computational resources required for this procedure are often prohibitively expensive. Furthermore, human oversight of the optimization routine is often required to determine whether the routine ran into any problems and if it successfully converged. Human oversight of estimation for each bootstrapped problem is usually not feasible.

A more reasonable alternative is a parametric bootstrap procedure:

1. Construct a large number of draws from the estimated joint distribution of parameters.
2. Compute the implied mean utility,  $\delta$ , and shares,  $s$ , for each draw. If a supply side was estimated, also computed the implied marginal costs,  $c$ , and prices,  $p$ .
3. Compute the desired post-estimation output under each of these parametric bootstrap samples. Again, from the resulting empirical distribution, construct bootstrap confidence intervals.

Compared to the standard bootstrap procedure, the parametric bootstrap requires far fewer computational resources, and is simple enough to not require human oversight of each bootstrap iteration. The primary complication to this procedure is that when supply is estimated, equilibrium prices and shares need to be computed for each parametric bootstrap sample by iterating over the  $\zeta$ -markup equation from *Morrow and Skerlos (2011)*. Although nontrivial, this fixed point iteration problem is much less demanding than the full optimization routine required to solve the BLP problem from the start.

An empirical distribution of results computed according to this parametric bootstrap procedure can be created with the `ProblemResults.bootstrap` method, which returns a `BootstrappedResults` class that can be used just like `ProblemResults` to compute various post-estimation outputs. The difference is that `BootstrappedResults` methods return arrays with an extra first dimension, along which bootstrapped results are stacked.

We'll construct 90% parametric bootstrap confidence intervals for estimated mean own elasticities in each market of the fake cereal problem. Usually, bootstrapped confidence intervals should be based on thousands of draws, but we'll only use a few for the sake of speed in this example.

```
bootstrapped_results = results.bootstrap(draws=100, seed=0)
bootstrapped_results
```

```
Bootstrapped Results Summary:
```

```
=====
Computation  Bootstrap
   Time      Draws
-----  -----
00:00:38     100
=====
```

```
bounds = np.percentile(
    bootstrapped_results.extract_diagonal_means(
        bootstrapped_results.compute_elasticities()
    ),
    q=[10, 90],
    axis=0
)
table = pd.DataFrame(index=problem.unique_market_ids, data={
    'Lower Bound': bounds[0].flatten(),
    'Mean Own Elasticity': aggregates.flatten(),
    'Upper Bound': bounds[1].flatten()
})
table.round(2).head()
```

	Lower Bound	Mean Own Elasticity	Upper Bound
C01Q1	-14.06	-0.81	10.66
C01Q2	-8.97	-0.69	7.54
C03Q1	-11.64	-0.55	7.98
C03Q2	-14.46	-0.60	6.47
C04Q1	-12.00	-0.68	5.61

## 4.4.6 Optimal Instruments

Given a consistent estimate of  $\theta$ , we may want to compute the optimal instruments of *Chamberlain (1987)* and use them to re-solve the problem. Optimal instruments have been shown, for example, by *Reynaert and Verboven (2014)*, to reduce bias, improve efficiency, and enhance stability of BLP estimates.

The `ProblemResults.compute_optimal_instruments` method computes the expected Jacobians that comprise the optimal instruments by integrating over the density of  $\xi$  (and  $\omega$  if a supply side was estimated). By default, the method approximates this integral by averaging over the Jacobian realizations computed under draws from the asymptotic normal distribution of the error terms. Since this process is computationally expensive and often doesn't make much of a difference, we'll use `method='approximate'` in this example to simply evaluate the Jacobians at the expected value of  $\xi$ , zero.

```
instrument_results = results.compute_optimal_instruments(method='approximate')
instrument_results
```

```
Optimal Instrument Results Summary:
```

```
=====
Computation Error Term
  Time      Draws
-----
00:00:01      1
=====
```

We can use the `OptimalInstrumentResults.to_problem` method to re-create the fake cereal problem with the estimated optimal excluded instruments.

```
updated_problem = instrument_results.to_problem()
updated_problem
```

```
Dimensions:
```

```
=====
  T      N      F      I      K1      K2      D      MD      ED
-----
94     2256     5     1880     1      4      4     14     1
=====
```

```
Formulations:
```

```
=====
      Column Indices:      0      1      2      3
-----
X1: Linear Characteristics  prices
X2: Nonlinear Characteristics  1      prices      sugar      mushy
      d: Demographics      income      income_squared      age      child
=====
```

We can solve this updated problem just like the original one. We'll start at our consistent estimate of  $\theta$ .



```

updated_results = updated_problem.solve(
    results.sigma,
    results.pi,
    optimization=pyblp.Optimization('bfgs', {'gtol': 1e-5}),
    method='1s'
)
updated_results

```

## Problem Results Summary:

```

=====
GMM   Objective  Gradient      Hessian      Hessian      Clipped  Weighting Matrix  Covariance Matrix
Step  Value       Norm         Min Eigenvalue Max Eigenvalue Shares   Condition Number  Condition Number
-----
1     +8.0E-14   +3.0E-06    +1.6E-04     +2.9E+04     0       +7.8E+07          +1.8E+08
=====

```

## Cumulative Statistics:

```

=====
Computation  Optimizer  Optimization  Objective  Fixed Point  Contraction
Time         Converged  Iterations    Evaluations Iterations    Evaluations
-----
00:02:11    Yes       42            50         45102        139676
=====

```

## Nonlinear Coefficient Estimates (Robust SEs in Parentheses):

```

=====
Sigma:      1      prices      sugar      mushy      |      Pi:      income      income_squared      age      child
-----
1           +2.1E-01      |      1           +6.0E+00      +0.0E+00      +1.6E-01      +0.0E+00
           (+7.8E-02)      |      |           (+5.2E-01)      |           (+2.0E-01)
prices      +0.0E+00      +3.0E+00      |      prices      +9.8E+01      -5.6E+00      +0.0E+00      +4.1E+00
           (+6.5E-01)      |      |           (+8.6E+01)      (+4.5E+00)      |           (+2.2E+00)
sugar       +0.0E+00      +0.0E+00      +2.7E-02      |      sugar       -3.1E-01      +0.0E+00      +4.9E-02      +0.0E+00
           (+7.2E-03)      |      |           (+3.5E-02)      |           (+1.3E-02)
mushy       +0.0E+00      +0.0E+00      +0.0E+00      +3.0E-01      |      mushy       +9.7E-01      +0.0E+00      -5.4E-01      +0.0E+00
           (+1.0E-01)      |      |           (+2.9E-01)      |           (+1.8E-01)
=====

```

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Beta Estimates (Robust SEs in Parentheses):

```
=====
prices
-----
-3.1E+01
(+4.5E+00)
=====
```

---

The [online version](#) of the following section may be easier to read.

---

## 4.5 Problem Simulation Tutorial

```
import pyblp
import numpy as np
import pandas as pd

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

Before configuring and solving a problem with real data, it may be a good idea to perform Monte Carlo analysis on simulated data to verify that it is possible to accurately estimate model parameters. For example, before configuring and solving the example problems in the prior tutorials, it may have been a good idea to simulate data according to the assumed models of supply and demand. During such Monte Carlo analysis, the data would only be used to determine sample sizes and perhaps to choose reasonable true parameters.

Simulations are configured with the *Simulation* class, which requires many of the same inputs as *Problem*. The two main differences are:

1. Variables in formulations that cannot be loaded from `product_data` or `agent_data` will be drawn from independent uniform distributions.
2. True parameters and the distribution of unobserved product characteristics are specified.

First, we'll use *build\_id\_data* to build market and firm IDs for a model in which there are  $T = 50$  markets, and in each market  $t$ , a total of  $J_t = 20$  products produced by  $F = 10$  firms.

```
id_data = pyblp.build_id_data(T=50, J=20, F=10)
```

Next, we'll create an *Integration* configuration to build agent data according to a Gauss-Hermite product rule that exactly integrates polynomials of degree  $2 \times 9 - 1 = 17$  or less.

```
integration = pyblp.Integration('product', 9)
integration
```

```
Configured to construct nodes and weights according to the level-9 Gauss-Hermite product rule with options {}.
```

We'll then pass these data to *Simulation*. We'll use *Formulation* configurations to create an  $X_1$  that consists of a constant, prices, and an exogenous characteristic; an  $X_2$  that consists only of the same exogenous characteristic; and an  $X_3$  that consists of the common exogenous characteristic and a cost-shifter.

```
simulation = pyblp.Simulation(
    product_formulations=(
        pyblp.Formulation('1 + prices + x'),
        pyblp.Formulation('0 + x'),
        pyblp.Formulation('0 + x + z')
    ),
    beta=[1, -2, 2],
    sigma=1,
    gamma=[1, 4],
    product_data=id_data,
    integration=integration,
    seed=0
)
simulation
```

Dimensions:

```
=====
T      N      F      I      K1      K2      K3
-----
50    1000    10     450     3        1        2
=====
```

Formulations:

```
=====
Column Indices:      0      1      2
-----
X1: Linear Characteristics      1  prices  x
X2: Nonlinear Characteristics   x
X3: Linear Cost Characteristics  x      z
=====
```

Nonlinear Coefficient True Values:

```
=====
Sigma:      x
-----
x      +1.0E+00
=====
```

Beta True Values:

```
=====
1      prices      x
=====
```

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```

-----
+1.0E+00  -2.0E+00  +2.0E+00
=====

Gamma True Values:
=====
      x          z
-----
+1.0E+00  +4.0E+00
=====

```

When *Simulation* is initialized, it constructs *Simulation.agent\_data* and simulates *Simulation.product\_data*.

The *Simulation* can be further configured with other arguments that determine how unobserved product characteristics are simulated and how marginal costs are specified.

At this stage, simulated variables are not consistent with true parameters, so we still need to solve the simulation with *Simulation.replace\_endogenous*. This method replaced simulated prices and market shares with values that are consistent with the true parameters. Just like *ProblemResults.compute\_prices*, to do so it iterates over the  $\zeta$ -markup equation from *Morrow and Skerlos (2011)*.

```

simulation_results = simulation.replace_endogenous()
simulation_results

```

```

Simulation Results Summary:
=====
Computation  Fixed Point  Contraction
   Time      Iterations  Evaluations
-----
 00:00:01      721          721
=====

```

Now, we can try to recover the true parameters by creating and solving a *Problem*.

The convenience method *SimulationResults.to\_problem* constructs some basic “sums of characteristics” BLP instruments that are functions of all exogenous numerical variables in the problem. In this example, excluded demand-side instruments are the cost-shifter  $z$  and traditional BLP instruments constructed from  $x$ . Excluded supply-side instruments are traditional BLP instruments constructed from  $x$  and  $z$ .

```

problem = simulation_results.to_problem()
problem

```

```

Dimensions:
=====

```

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```

T      N      F      I      K1      K2      K3      MD      MS
-----
50    1000    10    450    3      1      2      5      6
=====

Formulations:
=====
      Column Indices:      0      1      2
-----
X1: Linear Characteristics      1  prices  x
X2: Nonlinear Characteristics    x
X3: Linear Cost Characteristics  x      z
=====

```

We'll choose starting values that are half the true parameters so that the optimization routine has to do some work. Note that since we're jointly estimating the supply side, we need to provide an initial value for the linear coefficient on prices because this parameter cannot be concentrated out of the problem (unlike linear coefficients on exogenous characteristics).

```

results = problem.solve(
    sigma=0.5 * simulation.sigma,
    pi=0.5 * simulation.pi,
    beta=[None, 0.5 * simulation.beta[1], None],
    optimization=pyblp.Optimization('l-bfgs-b', {'gtol': 1e-5})
)
results

```

Problem Results Summary:

```

=====
GMM   Objective   Projected   Reduced Hessian   Reduced Hessian   Clipped   Weighting Matrix   Covariance Matrix
Step  Value        Gradient Norm Min Eigenvalue   Max Eigenvalue   Shares   Condition Number   Condition Number
-----
  2   +6.4E+00     +6.9E-08   +7.2E+00         +3.8E+03         0        +3.7E+04          +1.5E+04
=====

```

Cumulative Statistics:

```

=====
Computation   Optimizer   Optimization   Objective   Fixed Point   Contraction
Time          Converged   Iterations     Evaluations  Iterations    Evaluations
-----
00:00:33     Yes        23             30          7693         24410

```

(continues on next page)

```
=====
Nonlinear Coefficient Estimates (Robust SEs in Parentheses):
```

```
=====
Sigma:      x
-----
x          +7.8E-01
          (+5.2E-01)
=====
```

```
Beta Estimates (Robust SEs in Parentheses):
```

```
=====
      1          prices          x
-----
+9.6E-01  -2.0E+00  +2.1E+00
(+9.3E-02) (+2.4E-02) (+1.4E-01)
=====
```

```
Gamma Estimates (Robust SEs in Parentheses):
```

```
=====
      x          z
-----
+9.8E-01  +4.0E+00
(+8.7E-02) (+8.5E-02)
=====
```

The parameters seem to have been estimated reasonably well.

```
np.c_[simulation.beta, results.beta]
```

```
array([[ 1.          ,  0.96223514],
       [-2.          , -2.00792431],
       [ 2.          ,  2.10032015]])
```

```
np.c_[simulation.gamma, results.gamma]
```

```
array([[1.          ,  0.97820624],
       [4.          ,  4.03121577]])
```

```
np.c_[simulation.sigma, results.sigma]
```

```
array([[1.          , 0.78358853]])
```

In addition to checking that the configuration for a model based on actual data makes sense, the *Simulation* class can also be a helpful tool for better understanding under what general conditions BLP models can be accurately estimated. Simulations are also used extensively in pyblp's test suite.



## API DOCUMENTATION

The majority of the package consists of classes, which compartmentalize different aspects of the BLP model. There are some convenience functions as well.

### 5.1 Configuration Classes

Various components of the package require configurations for how to approximate integrals, solve fixed point problems, and solve optimization problems. Such configurations are specified with the following classes.

<i>Formulation</i> (formula[, absorb, ...])	Configuration for designing matrices and absorbing fixed effects.
<i>Integration</i> (specification, size[, ...])	Configuration for building integration nodes and weights.
<i>Iteration</i> (method[, method_options, ...])	Configuration for solving fixed point problems.
<i>Optimization</i> (method[, method_options, ...])	Configuration for solving optimization problems.

#### 5.1.1 pyblp.Formulation

**class** `pyblp.Formulation` (*formula*, *absorb=None*, *absorb\_method=None*, *absorb\_options=None*)  
Configuration for designing matrices and absorbing fixed effects.

Internally, the `patsy` package is used to convert data and R-style formulas into matrices. All of the standard `binary operators` can be used to design complex matrices of factor interactions:

- `+` - Set union of terms.
- `-` - Set difference of terms.
- `*` - Short-hand. The formula `a * b` is the same as `a + b + a:b`.
- `/` - Short-hand. The formula `a / b` is the same as `a + a:b`.
- `:` - Interactions between two sets of terms.
- `**` - Interactions up to an integer degree.

However, since factors need to be differentiated (for example, when computing elasticities), only the most essential functions are supported:

- `C` - Mark a variable as categorical. See `patsy.builtins.C()`. Arguments are not supported.
- `I` - Encapsulate mathematical operations. See `patsy.builtins.I()`.
- `log` - Natural logarithm function.

- `exp` - Natural exponential function.

Data associated with variables should generally already be transformed. However, when encapsulated by `I()`, these operators function like normal mathematical operators on numeric variables: `+` adds, `-` subtracts, `*` multiplies, `/` divides, and `**` exponentiates.

Internally, mathematical operations are parsed and evaluated by the `SymPy` package, which is also used to symbolically differentiate terms when derivatives are needed.

### Parameters

- **`formula`** (*str*) – R-style formula used to design a matrix. Variable names will be validated when this formulation and data are passed to a function that uses them. By default, an intercept is included, which can be removed with `0` or `-1`. If `absorb` is specified, intercepts are ignored.
- **`absorb`** (*str, optional*) – R-style formula used to design a matrix of categorical variables representing fixed effects, which will be absorbed into the matrix designed by `formula` by the `PyHDFE` package. Fixed effect absorption is only supported for some matrices. Unlike `formula`, intercepts are ignored. Only categorical variables are supported.
- **`absorb_method`** (*str, optional*) – Method by which fixed effects will be absorbed. For a full list of supported methods, refer to the `residualize_method` argument of `pyhdfe.create()`.

By default, the simplest methods are used: simple de-meaning for a single fixed effect and simple iterative de-meaning by way of the method of alternating projections (MAP) for multiple dimensions of fixed effects. For multiple dimensions, non-accelerated MAP is unlikely to be the fastest algorithm. If fixed effect absorption seems to be taking a long time, consider using a different method such as `'lsnr'`, using `absorb_options` to specify a MAP acceleration method, or configuring other options such as termination tolerances.

- **`absorb_options`** (*dict, optional*) – Configuration options for the chosen method, which will be passed to the `options` argument of `pyhdfe.create()`.

### Examples

---

The [online version](#) of the following section may be easier to read.

---

### Formulation Example

```
import pyblp

pyblp.__version__
'0.10.1'
```

In this example, we'll design a matrix without an intercept, but with both prices and another numeric size variable.

```
formulation = pyblp.Formulation('0 + prices + size')
formulation

prices + size
```

Next, we'll design a second matrix with an intercept, with first- and second-degree size terms, with categorical product IDs and years, and with the interaction of the last two. The first formulation will include the fixed effects as indicator variables, and the second will absorb them.

```
formulation1 = pyblp.Formulation('size + I(size ** 2) + C(product) * C(year)')
formulation1

1 + size + I(size ** 2) + C(product) + C(year) + C(product):C(year)
```

```
formulation2 = pyblp.Formulation('size + I(size ** 2)', absorb='C(product) * C(year)')
formulation2

size + I(size ** 2) + Absorb[C(product)] + Absorb[C(year)] + Absorb[C(product):C(year)]
```

Finally, we'll design a third matrix with an intercept and with a yearly trend interacted with the natural logarithm of income and categorical education. Absorption of continuous variables is not supported, so we need to use dummy variables.

```
formulation = pyblp.Formulation('year:(log(income) + C(education))')
formulation

1 + year:log(income) + year:C(education)
```

## 5.1.2 pyblp.Integration

**class** `pyblp.Integration` (*specification, size, specification\_options=None*)  
Configuration for building integration nodes and weights.

### Parameters

- **specification** (*str*) – How to build nodes and weights. One of the following:
  - 'monte\_carlo' - Draw from a pseudo-random standard multivariate normal distribution. Integration weights are  $1 / \text{size}$ . The `seed` field of `options` can be used to seed the random number generator.
  - 'halton' - Generate nodes according to the Halton. A different prime (starting with 2, 3, 5, etc.) is used for each dimension of integration. To eliminate correlation between dimensions, the first 1000 values are by default discarded in each dimension. To further improve performance (particularly in settings with many dimensions), sequences are also by default scrambled with the algorithm of [Owen \(2017\)](#). The `discard`, `scramble`, and `seed` fields of `options` can be used to configure these default settings.
  - 'lhs' - Generate nodes according to Latin Hypercube Sampling (LHS). Integration weights are  $1 / \text{size}$ . The `seed` field of `options` can be used to seed the random number generator.
  - 'mlhs' - Generate nodes according to Modified Latin Hypercube Sampling (MLHS) described by [Hess, Train, and Polak \(2004\)](#). Integration weights are  $1 / \text{size}$ . The `seed` field of `options` can be used to seed the random number generator.
  - 'product' - Generate nodes and weights according to the level-size Gauss-Hermite product rule.
  - 'nested\_product' - Generate nodes and weights according to the level-size nested Gauss-Hermite product rule. Weights can be negative.
  - 'grid' - Generate a sparse grid of nodes and weights according to the level-size Gauss-Hermite quadrature rule. Weights can be negative.
  - 'nested\_grid' - Generate a sparse grid of nodes and weights according to the level size nested Gauss-Hermite quadrature rule. Weights can be negative.

Best practice for low dimensions is probably to use 'product' to a relatively high degree of polynomial accuracy. In higher dimensions, 'grid' or 'halton' appears to scale the best. For more information, see [Judd and Skrainka \(2011\)](#) and [Conlon and Gortmaker \(2020\)](#).

Sparse grids are constructed in analogously to the Matlab function `nwspgr` created by Florian Heiss and Viktor Winschel. For more information, see [Heiss and Winschel \(2008\)](#).

- **size** (*int*) – The number of draws if `specification` is 'monte\_carlo', 'halton', 'lhs', or 'mlhs', and the level of the quadrature rule otherwise.
- **specification\_options** (*dict, optional*) – Options for the integration specification. The 'monte\_carlo', 'halton', 'lhs', and 'mlhs' specifications support the following option:
  - **seed** : (*int*) - Passed to `numpy.random.RandomState` to seed the random number generator before building integration nodes. By default, a seed is not passed to the random number generator. For 'halton' draws, this is only relevant if `scramble` is `True` (which is the default).

The 'halton' specification supports the following options:

- **discard** : (*int*) - How many values at the beginning of each dimension's Halton sequence to discard. Discarding values at the start of each dimension's sequence is the simplest way to eliminate correlation between dimensions. By default, the first 1000 values in each dimension are discarded.
- **scramble** : (*bool*) - Whether to scramble the sequences with the algorithm of *Owen (2017)*. By default, sequences are scrambled.

## Examples

---

The [online version](#) of the following section may be easier to read.

---

## Integration Example

```
import pyblp

pyblp.__version__

'0.10.1'
```

In this example, we'll build a Monte Carlo configuration with 1,000 draws for each market and a fixed seed.

```
integration = pyblp.Integration('monte_carlo', size=1000, specification_options={'seed': 0})
integration
```

```
Configured to construct nodes and weights with Monte Carlo simulation with options {seed: 0}.
```

Depending on the dimension of the integration problem, a level six sparse grid configuration may have a similar number of nodes. However, even if there are fewer nodes, it is likely to perform better in the BLP problem. Sparse grid construction is deterministic, so a seed is not needed to fix the grid every time we use this configuration.

```
integration = pyblp.Integration('grid', size=7)
integration
```

```
Configured to construct nodes and weights in a sparse grid according to the level-7 Gauss-Hermite rule with options {}.
```

### 5.1.3 pyblp.Iteration

**class** `pyblp.Iteration` (*method*, *method\_options=None*, *compute\_jacobian=False*)  
 Configuration for solving fixed point problems.

#### Parameters

- **method** (*str or callable*) – The fixed point iteration routine that will be used. The following routines do not use analytic Jacobians:
  - 'simple' - Non-accelerated iteration.
  - 'squarem' - SQUAREM acceleration method of *Varadhan and Roland (2008)* and considered in the context of the BLP problem in *Reynaerts, Varadhan, and Nash (2012)*. This implementation uses a first-order squared non-monotone extrapolation scheme. If there are any errors during the acceleration step, it uses the last values for the next iteration of the algorithm.
  - 'broyden1' - Use the `scipy.optimize.root()` Broyden's first Jacobian approximation method, known as Broyden's good method.
  - 'broyden2' - Use the `scipy.optimize.root()` Broyden's second Jacobian approximation method, known as Broyden's bad method.
  - 'anderson' - Use the `scipy.optimize.root()` Anderson method.
  - 'krylov' - Use the `scipy.optimize.root()` Krylov approximation for inverse Jacobian method.
  - 'diagbroyden' - Use the `scipy.optimize.root()` diagonal Broyden Jacobian approximation method.
  - 'df-sane' - Use the `scipy.optimize.root()` derivative-free spectral method.

The following routines can use analytic Jacobians:

- 'hybr' - Use the `scipy.optimize.root()` modification of the Powell hybrid method implemented in MINIPACK.
- 'lm' - Uses the `scipy.optimize.root()` modification of the Levenberg-Marquardt algorithm implemented in MINIPACK.

The following trivial routine can be used to simply return the initial values:

- 'return' - Assume that the initial values are the optimal ones.

Also accepted is a custom callable method with the following form:

```
method(initial, contraction, callback, **options) -> (final,
↳ converged)
```

where `initial` is an array of initial values, `contraction` is a callable contraction mapping of the form specified below, `callback` is a function that should be called without any arguments after each major iteration (it is used to record the number of major iterations), `options` are specified below, `final` is an array of final values, and `converged` is a flag for whether the routine converged.

The `contraction` function has the following form:

```
contraction(x0) -> (x1, weights, jacobian)
```

where `weights` are either `None` or a vector of weights that should multiply `x1 - x` before computing the norm of the differences, and `jacobian` is `None` if `compute_jacobian` is `False`.

Regardless of the chosen routine, if there are any computational issues that create infinities or null values, `final` will be the second to last iteration's values.

- **method\_options** (*dict, optional*) – Options for the fixed point iteration routine.

For routines other than `'simple'`, `'squarem'`, and `'return'`, these options will be passed to `options` in `scipy.optimize.root()`. Refer to the SciPy documentation for information about which options are available. By default, the `tol_norm` option is configured to use the infinity norm for SciPy methods other than `'hybr'` and `'lm'`, for which a norm cannot be specified.

The `'simple'` and `'squarem'` methods support the following options:

- **max\_evaluations** : (*int*) - Maximum number of contraction mapping evaluations. The default value is 5000.
- **atol** : (*float*) - Absolute tolerance for convergence of the configured norm. The default value is  $1e-14$ . To use only a relative tolerance, set this to zero.
- **rtol** (*float*) - Relative tolerance for convergence of the configured norm. The default value is zero; that is, only absolute tolerance is used by default.
- **norm** : (*callable*) - The norm to be used. By default, the  $\ell^\infty$ -norm is used. If specified, this should be a function that accepts an array of differences and that returns a scalar norm.

The `'squarem'` routine accepts additional options that mirror those in the [SQUAREM](#) package, written in R by Ravi Varadhan, which identifies the step length with  $-\alpha$  from [Varadhan and Roland \(2008\)](#):

- **scheme** : (*int*) - The default value is 3, which corresponds to S3 in [Varadhan and Roland \(2008\)](#). Other acceptable schemes are 1 and 2, which correspond to S1 and S2.
  - **step\_min** : (*float*) - The initial value for the minimum step length. The default value is 1.0.
  - **step\_max** : (*float*) - The initial value for the maximum step length. The default value is 1.0.
  - **step\_factor** : (*float*) - When the step length exceeds `step_max`, it is set equal to `step_max`, but `step_max` is scaled by this factor. Similarly, if `step_min` is negative and the step length is below `step_min`, it is set equal to `step_min` and `step_min` is scaled by this factor. The default value is 4.0.
- **compute\_jacobian** (*bool, optional*) – Whether to compute an analytic Jacobian during iteration, which must be `False` if `method` does not use analytic Jacobians. By default, analytic Jacobians are not computed, and if a `method` is selected that supports analytic Jacobians, they will by default be numerically approximated.

## Examples



---

The [online version](#) of the following section may be easier to read.

---

### Iteration Example

```
import pyblp
import numpy as np

pyblp.__version__

'0.10.1'
```

In this example, we'll build a SQUAREM configuration with a  $\ell^2$ -norm and use scheme S1 from *Varadhan and Roland (2008)*.

```
iteration = pyblp.Iteration('squarem', {'norm': np.linalg.norm, 'scheme': 1})
iteration
```

```
Configured to iterate using the SQUAREM acceleration method without analytic Jacobians with options {atol: +1.000000E-
↪14, rtol: 0, max_evaluations: 5000, norm: numpy.linalg.norm, scheme: 1, step_min: +1.000000E+00, step_max: +1.
↪000000E+00, step_factor: +4.000000E+00}.
```

Next, instead of using a built-in routine, we'll create a custom method that implements a version of simple iteration, which, for the sake of having a nontrivial example, arbitrarily identifies a major iteration with three objective evaluations.

```
def custom_method(initial, contraction, callback, max_evaluations, tol, norm):
    x = initial
    evaluations = 0
    while evaluations < max_evaluations:
        x0, (x, weights, _) = x, contraction(x)
        evaluations += 1
        if evaluations % 3 == 0:
            callback()
        if weights is None:
            difference = norm(x - x0)
        else:
            difference = norm(weights * (x - x0))
        if difference < tol:
```

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```
        break
    return x, evaluations < max_evaluations
```

We can then use this custom method to build a custom iteration configuration.

```
iteration = pyblp.Iteration(custom_method)
iteration
```

```
Configured to iterate using a custom method without analytic Jacobians with options {}.
```

### 5.1.4 pyblp.Optimization

`class pyblp.Optimization` (*method*, *method\_options=None*, *compute\_gradient=True*, *universal\_display=True*)

Configuration for solving optimization problems.

#### Parameters

- **method** (*str or callable*) – The optimization routine that will be used. The following routines support parameter bounds and use analytic gradients:
  - 'knitro' - Uses an installed version of [Artleys Knitro](#). Python 3 is supported by Knitro version 10.3 and newer. A number of environment variables most likely need to be configured properly, such as `KNITRODIR`, `ARTELYS_LICENSE`, `LD_LIBRARY_PATH` (on Linux), and `DYLD_LIBRARY_PATH` (on Mac OS X). For more information, refer to the [Knitro installation guide](#).
  - 'slsqp' - Uses the `scipy.optimize.minimize()` SLSQP routine.
  - 'trust-constr' - Uses the `scipy.optimize.minimize()` trust-region routine.
  - 'l-bfgs-b' - Uses the `scipy.optimize.minimize()` L-BFGS-B routine.
  - 'tnc' - Uses the `scipy.optimize.minimize()` TNC routine.

The following routines also use analytic gradients but will ignore parameter bounds (not bounding the problem may create issues if the optimizer tries out large parameter values that create overflow errors):

- 'cg' - Uses the `scipy.optimize.minimize()` CG routine.
- 'bfgs' - Uses the `scipy.optimize.minimize()` BFGS routine.
- 'newton-cg' - Uses the `scipy.optimize.minimize()` Newton-CG routine.

The following routines do not use analytic gradients and will also ignore parameter bounds (without analytic gradients, optimization will likely be much slower):

- 'nelder-mead' - Uses the `scipy.optimize.minimize()` Nelder-Mead routine.
- 'powell' - Uses the `scipy.optimize.minimize()` Powell routine.

The following trivial routine can be used to evaluate an objective at specific parameter values:

- 'return' - Assume that the initial parameter values are the optimal ones.

Also accepted is a custom callable method with the following form:

```
method(initial, bounds, objective_function, iteration_callback,
↳ **options) -> (final, converged)
```

where `initial` is an array of initial parameter values, `bounds` is a list of (min, max) pairs for each element in `initial`, `objective_function` is a callable objective function of the form specified below, `iteration_callback` is a function that should be called without any arguments after each major iteration (it is used to record the number of major iterations), `options` are specified below, `final` is an array of optimized parameter values, and `converged` is a flag for whether the routine converged.

The `objective_function` has the following form:

```
objective_function(theta) -> (objective, gradient)
```

where `gradient` is `None` if `compute_gradient` is `False`.

- **method\_options** (*dict, optional*) – Options for the optimization routine.

For any non-custom method other than 'knitro' and 'return', these options will be passed to options in `scipy.optimize.minimize()`. Refer to the SciPy documentation for information about which options are available for each optimization routine.

If method is 'knitro', these options should be [Knitro user options](#). The non-standard `knitro_dir` option can also be specified. The following options have non-standard default values:

- **knitro\_dir** : (*str*) - By default, the KNITRODIR environment variable is used. Otherwise, this option should point to the installation directory of Knitro, which contains direct subdirectories such as 'examples' and 'lib'. For example, on Windows this option could be '/Program Files/Artleys3/Knitro 10.3.0'.
  - **algorithm** : (*int*) - The optimization algorithm to be used. The default value is 1, which corresponds to the Interior/Direct algorithm.
  - **gradopt** : (*int*) - How the objective's gradient is computed. The default value is 1 if `compute_gradient` is `True` and is 2 otherwise, which corresponds to estimating the gradient with finite differences.
  - **hessopt** : (*int*) - How the objective's Hessian is computed. The default value is 2, which corresponds to computing a quasi-Newton BFGS Hessian.
  - **honorbnds** : (*int*) - Whether to enforce satisfaction of simple variable bounds. The default value is 1, which corresponds to enforcing that the initial point and all subsequent solution estimates satisfy the bounds.
- **compute\_gradient** (*bool, optional*) – Whether to compute an analytic objective gradient during optimization, which must be `False` if method does not use analytic gradients, and must be `True` if method is 'newton-cg', which requires an analytic gradient.  
  
By default, analytic gradients are computed. Not using an analytic gradient will likely slow down estimation a good deal. If `False`, an analytic gradient may still be computed once at the end of optimization to compute optimization results. To always use finite differences, `finite_differences` in `Problem.solve()` can be set to `True`.
  - **universal\_display** (*bool, optional*) – Whether to format optimization progress such that the display looks the same for all routines. By default, the universal display is used and some `method_options` are used to prevent default displays from showing up.

## Examples

---

The [online version](#) of the following section may be easier to read.

---

## Optimization Example

```
import pyblp
import numpy as np

pyblp.__version__
'0.10.1'
```

In this example, we'll build a L-BFGS-B configuration with a non-default tolerance.

```
optimization = pyblp.Optimization('l-bfgs-b', {'gtol': 1e-3})
optimization
```

```
Configured to optimize using the L-BFGS-B algorithm implemented in SciPy with analytic gradients and options {gtol: +1.
↪000000E-03}.
```

Next, instead of using a non-custom routine, we'll create a custom method that implements a grid search over parameter values between specified bounds.

```
from itertools import product
def custom_method(initial, bounds, objective_function, iteration_callback):
    best_values = initial
    best_objective = np.inf
    for values in product(*(np.linspace(l, u, 10) for l, u in bounds)):
        objective, _ = objective_function(values)
        if objective < best_objective:
            best_values = values
            best_objective = objective
        iteration_callback()
    return best_values, True
```

We can then use this custom method to build an optimization configuration.

```
optimization = pyblp.Optimization(custom_method, compute_gradient=False)
optimization
```

```
Configured to optimize using a custom method without analytic gradients and options {}.
```

## 5.2 Data Construction Functions

There are also a number of convenience functions that can be used to construct common components of product and agent data.

<code>build_matrix(formulation, data)</code>	Construct a matrix according to a formulation.
<code>build_blp_instruments(formulation, product_data)</code>	Construct “sums of characteristics” excluded BLP instruments.
<code>build_differentiation_instruments(...[, ...])</code>	Construct excluded differentiation instruments.
<code>build_id_data(T, J, F)</code>	Build a balanced panel of market and firm IDs.
<code>build_ownership(product_data[, ...])</code>	Build ownership matrices, $O$ .
<code>build_integration(integration, dimensions)</code>	Build nodes and weights for integration over agent choice probabilities.
<code>data_to_dict(data[, ignore_empty])</code>	Convert a NumPy record array into a dictionary.

### 5.2.1 pyblp.build\_matrix

`pyblp.build_matrix(formulation, data)`

Construct a matrix according to a formulation.

#### Parameters

- **formulation** (*Formulation*) – *Formulation* configuration for the matrix. Variable names should correspond to fields in `data`. The `absorb` argument of *Formulation* can be used to absorb fixed effects after the matrix has been constructed.
- **data** (*structured array-like*) – Fields can be used as variables in `formulation`.

**Returns** The built matrix.

**Return type** `ndarray`

#### Examples

---

The [online version](#) of the following section may be easier to read.

---

## Building a Matrix Example

```
import pyblp
import pandas as pd
```

```
pyblp.__version__
```

```
'0.10.1'
```

In this example, we'll load the fake cereal data from *Nevo (2000)* and create a simple matrix involving a constant, prices, and shares.

```
formulation = pyblp.Formulation('1 + prices + shares')
```

```
formulation
```

```
1 + prices + shares
```

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
```

```
product_data.head()
```

	market_ids	city_ids	quarter	product_ids	firm_ids	brand_ids	shares	\
0	C01Q1	1	1	F1B04	1	4	0.012417	
1	C01Q1	1	1	F1B06	1	6	0.007809	
2	C01Q1	1	1	F1B07	1	7	0.012995	
3	C01Q1	1	1	F1B09	1	9	0.005770	
4	C01Q1	1	1	F1B11	1	11	0.017934	

	prices	sugar	mushy	...	demand_instruments10	demand_instruments11	\
0	0.072088	2	1	...	2.116358	-0.154708	
1	0.114178	18	1	...	-7.374091	-0.576412	
2	0.132391	4	1	...	2.187872	-0.207346	
3	0.130344	3	0	...	2.704576	0.040748	
4	0.154823	12	0	...	1.261242	0.034836	

	demand_instruments12	demand_instruments13	demand_instruments14	\
0	-0.005796	0.014538	0.126244	
1	0.012991	0.076143	0.029736	

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```

2          0.003509          0.091781          0.163773
3         -0.003724          0.094732          0.135274
4         -0.000568          0.102451          0.130640

demand_instruments15 demand_instruments16 demand_instruments17 \
0          0.067345          0.068423          0.034800
1          0.087867          0.110501          0.087784
2          0.111881          0.108226          0.086439
3          0.088090          0.101767          0.101777
4          0.084818          0.101075          0.125169

demand_instruments18 demand_instruments19
0          0.126346          0.035484
1          0.049872          0.072579
2          0.122347          0.101842
3          0.110741          0.104332
4          0.133464          0.121111

[5 rows x 30 columns]

```

```

matrix = pyblp.build_matrix(formulation, product_data)
matrix
array([[1.          , 0.07208794, 0.01241721],
       [1.          , 0.11417849, 0.00780939],
       [1.          , 0.13239066, 0.01299451],
       ...,
       [1.          , 0.13701741, 0.00222918],
       [1.          , 0.10017433, 0.01146267],
       [1.          , 0.12755747, 0.02620832]])

```

For various reasons, we may want to absorb fixed effects into the matrix. This can be done with the `absorb` argument of *Formulation*. We'll now re-create the matrix, absorbing product-specific fixed effects. Note that the constant column is now ignored.

```

absorb_formulation = pyblp.Formulation('prices + shares', absorb='product_ids')
absorb_formulation

prices + shares + Absorb[product_ids]

```

```
demeaned_matrix = pyblp.build_matrix(absorb_formulation, product_data)
demeaned_matrix
```

```
array([[ -0.01124832, -0.00052161],
       [ -0.00713476, -0.03144549],
       [  0.02367765, -0.01664996],
       ...,
       [  0.03371995, -0.00779841],
       [-0.00417404, -0.0117508 ],
       [-0.01195648,  0.00666695]])
```

## 5.2.2 pyblp.build\_blp\_instruments

`pyblp.build_blp_instruments` (*formulation, product\_data*)

Construct “sums of characteristics” excluded BLP instruments.

Traditional “sums of characteristics” BLP instruments are

$$Z^{\text{BLP}}(X) = [Z^{\text{BLP,Other}}(X), Z^{\text{BLP,Rival}}(X)], \quad (5.1)$$

in which  $X$  is a matrix of product characteristics,  $Z^{\text{BLP,Other}}(X)$  is a second matrix that consists of sums over characteristics of non-rival goods, and  $Z^{\text{BLP,Rival}}(X)$  is a third matrix that consists of sums over rival goods. All three matrices have the same dimensions.

---

**Note:** To construct simpler, firm-agnostic instruments that are sums over characteristics of other goods, specify a constant column of firm IDs and keep only the first half of the instrument columns.

---

Let  $x_{jt}$  be the vector of characteristics in  $X$  for product  $j$  in market  $t$ , which is produced by firm  $f$ . That is,  $j \in J_{ft}$ . Then,

$$\begin{aligned} Z_{jt}^{\text{BLP,Other}}(X) &= \sum_{k \in J_{ft} \setminus \{j\}} x_{kt}, \\ Z_{jt}^{\text{BLP,Rival}}(X) &= \sum_{k \notin J_{ft}} x_{kt}. \end{aligned} \quad (5.2)$$

---

**Note:** Usually, any supply or demand shifters are added to these excluded instruments, depending on whether they are meant to be used for demand- or supply-side estimation.

---

### Parameters

- **formulation** (*Formulation*) – *Formulation* configuration for  $X$ , the matrix of product characteristics used to build excluded instruments. Variable names should correspond to fields in `product_data`.
- **product\_data** (*structured array-like*) – Each row corresponds to a product. Markets can have differing numbers of products. The following fields are required:
  - **market\_ids** : (*object*) - IDs that associate products with markets.
  - **firm\_ids** : (*object*) - IDs that associate products with firms.

Along with `market_ids` and `firm_ids`, the names of any additional fields can be used as variables in `formulation`.

**Returns** Traditional “sums of characteristics” BLP instruments,  $Z^{\text{BLP}}(X)$ .

**Return type** *ndarray*

### Examples

---

The [online version](#) of the following section may be easier to read.

---

### Building “Sums of Characteristics” BLP Instruments Example

```
import pyblp
import numpy as np
import pandas as pd

np.set_printoptions(precision=3)
pyblp.__version__

'0.10.1'
```

In this example, we'll load the automobile product data from *Berry, Levinsohn, and Pakes (1995)* and show how to construct the included instruments from scratch.

```
product_data = pd.read_csv(pyblp.data.BLP_PRODUCTS_LOCATION)
product_data.head()
```

	market_ids	clustering_ids	car_ids	firm_ids	region	shares	prices	\
0	1971	AMGREM71	129	15	US	0.001051	4.935802	
1	1971	AMHORN71	130	15	US	0.000670	5.516049	
2	1971	AMJAVL71	132	15	US	0.000341	7.108642	
3	1971	AMMATA71	134	15	US	0.000522	6.839506	
4	1971	AMAMBS71	136	15	US	0.000442	8.928395	

	hpwt	air	mpd	...	supply_instruments2	supply_instruments3	\
0	0.528997	0	1.888146	...	0.0	1.705933	
1	0.494324	0	1.935989	...	0.0	1.680910	
2	0.467613	0	1.716799	...	0.0	1.801067	
3	0.426540	0	1.687871	...	0.0	1.818061	
4	0.452489	0	1.504286	...	0.0	1.933210	

	supply_instruments4	supply_instruments5	supply_instruments6	\
0	1.595656	87.0	-61.959985	
1	1.490295	87.0	-61.959985	
2	1.357703	87.0	-61.959985	
3	1.261347	87.0	-61.959985	
4	1.237365	87.0	-61.959985	

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```

supply_instruments7  supply_instruments8  supply_instruments9  \
0          0.0          46.060389          29.786989
1          0.0          46.060389          29.786989
2          0.0          46.060389          29.786989
3          0.0          46.060389          29.786989
4          0.0          46.060389          29.786989

supply_instruments10  supply_instruments11
0          0.0          1.888146
1          0.0          1.935989
2          0.0          1.716799
3          0.0          1.687871
4          0.0          1.504286

[5 rows x 33 columns]

```

```
product_data[[f'demand_instruments{i}' for i in range(8)]]
```

```

demand_instruments0  demand_instruments1  demand_instruments2  \
0          4.0          1.840967          0.0
1          4.0          1.875639          0.0
2          4.0          1.902350          0.0
3          4.0          1.943423          0.0
4          4.0          1.917475          0.0
...          ...          ...          ...
2212         2.0          0.826512          2.0
2213         2.0          0.776462          2.0
2214         0.0          0.000000          0.0
2215         1.0          0.693796          1.0
2216         1.0          0.814913          1.0

demand_instruments3  demand_instruments4  demand_instruments5  \
0          6.844945          87.0          44.555539
1          6.797102          87.0          44.555539
2          7.016291          87.0          44.555539
3          7.045220          87.0          44.555539
4          7.228805          87.0          44.555539
...          ...          ...          ...
2212         4.775577          128.0          57.660253

```

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```

2213      5.278269      128.0      57.660253
2214      0.000000      130.0      58.514393
2215      3.267500      129.0      57.363973
2216      3.016154      129.0      57.363973

```

```

      demand_instruments6 demand_instruments7
0          0.0          167.325082
1          0.0          167.325082
2          0.0          167.325082
3          0.0          167.325082
4          0.0          167.325082
...          ...          ...
2212         57.0         351.758942
2213         57.0         351.758942
2214         60.0         355.654808
2215         58.0         352.890000
2216         58.0         352.890000

```

```
[2217 rows x 8 columns]
```

```
product_data[[f'supply_instruments{i}' for i in range(12)]]
```

```

      supply_instruments0 supply_instruments1 supply_instruments2 \
0          4.0          -3.109718          0.0
1          4.0          -3.041927          0.0
2          4.0          -2.986377          0.0
3          4.0          -2.894442          0.0
4          4.0          -2.953498          0.0
...          ...          ...          ...
2212         2.0          -1.770401          2.0
2213         2.0          -1.892345          2.0
2214         0.0          0.000000          0.0
2215         1.0          -0.365578          1.0
2216         1.0          -0.204674          1.0

      supply_instruments3 supply_instruments4 supply_instruments5 \
0          1.705933          1.595656          87.0
1          1.680910          1.490295          87.0
2          1.801067          1.357703          87.0
3          1.818061          1.261347          87.0

```

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```

4          1.933210          1.237365          87.0
...          ...          ...          ...
2212       1.272566          0.511989          128.0
2213       1.483875          0.511989          128.0
2214       0.000000          0.000000          130.0
2215       0.955511          0.142876          129.0
2216       0.875469          0.089795          129.0

      supply_instruments6  supply_instruments7  supply_instruments8  \
0          -61.959985          0.0          46.060389
1          -61.959985          0.0          46.060389
2          -61.959985          0.0          46.060389
3          -61.959985          0.0          46.060389
4          -61.959985          0.0          46.060389
...          ...          ...          ...
2212      -104.631050          57.0          97.039220
2213      -104.631050          57.0          97.039220
2214      -106.327167          60.0          98.024103
2215      -106.783331          58.0          97.222743
2216      -106.783331          58.0          97.222743

      supply_instruments9  supply_instruments10  supply_instruments11
0          29.786989          0.0          1.888146
1          29.786989          0.0          1.935989
2          29.786989          0.0          1.716799
3          29.786989          0.0          1.687871
4          29.786989          0.0          1.504286
...          ...          ...          ...
2212       27.861181          38.0          2.639135
2213       27.861181          38.0          2.136442
2214       28.809765          0.0          3.518846
2215       28.407171          19.0          3.016154
2216       28.407171          19.0          3.267500

```

[2217 rows x 12 columns]

The demand-side “sums of characteristics” BLP instruments included in `product_data` can be built from scratch with the `build_blp_instruments` function.

```
demand_instruments = pyblp.build_blp_instruments(pyblp.Formulation('1 + hpwt + air + mpd'), product_data)
```

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```
demand_instruments
array([[ 4.    ,  1.841,  0.    , ..., 44.556,  0.    , 167.325],
       [ 4.    ,  1.876,  0.    , ..., 44.556,  0.    , 167.325],
       [ 4.    ,  1.902,  0.    , ..., 44.556,  0.    , 167.325],
       ...,
       [ 0.    ,  0.    ,  0.    , ..., 58.514, 60.    , 355.655],
       [ 1.    ,  0.694,  1.    , ..., 57.364, 58.    , 352.89 ],
       [ 1.    ,  0.815,  1.    , ..., 57.364, 58.    , 352.89 ]])
```

The supply-side instruments from the original paper are “sums of characteristics” BLP instruments as well, but also include a standalone mpd shifter. Because of collinearity issues, the “rival” instrument constructed from the `trend` variable is excluded, and only the “own” instrument is retained.

```
supply_instruments = np.c_[
    pyblp.build_blp_instruments(pyblp.Formulation('1 + log(hpwt) + air + log(mpg) + log(space)'), product_data),
    pyblp.build_blp_instruments(pyblp.Formulation('0 + trend'), product_data)[: , 0],
    product_data['mpd'],
]
supply_instruments
array([[ 4.    , -3.11 ,  0.    , ..., 29.787,  0.    ,  1.888],
       [ 4.    , -3.042,  0.    , ..., 29.787,  0.    ,  1.936],
       [ 4.    , -2.986,  0.    , ..., 29.787,  0.    ,  1.717],
       ...,
       [ 0.    ,  0.    ,  0.    , ..., 28.81 ,  0.    ,  3.519],
       [ 1.    , -0.366,  1.    , ..., 28.407, 19.    ,  3.016],
       [ 1.    , -0.205,  1.    , ..., 28.407, 19.    ,  3.268]])
```



### 5.2.3 pyblp.build\_differentiation\_instruments

`pyblp.build_differentiation_instruments` (*formulation*, *product\_data*, *version='local'*, *interact=False*)

Construct excluded differentiation instruments.

Differentiation instruments in the spirit of *Gandhi and Houde (2017)* are

$$Z^{\text{Diff}}(X) = [Z^{\text{Diff,Other}}(X), Z^{\text{Diff,Rival}}(X)], \quad (5.3)$$

in which  $X$  is a matrix of product characteristics,  $Z^{\text{Diff,Other}}(X)$  is a second matrix that consists of sums over functions of differences between non-rival goods, and  $Z^{\text{Diff,Rival}}(X)$  is a third matrix that consists of sums over rival goods. Without optional interaction terms, all three matrices have the same dimensions.

---

**Note:** To construct simpler, firm-agnostic instruments that are sums over functions of differences between all different goods, specify a constant column of firm IDs and keep only the first half of the instrument columns.

---

Let  $x_{jt\ell}$  be characteristic  $\ell$  in  $X$  for product  $j$  in market  $t$ , which is produced by firm  $f$ . That is,  $j \in J_{ft}$ . Then in the “local” version of  $Z^{\text{Diff}}(X)$ ,

$$\begin{aligned} Z_{jt\ell}^{\text{Local,Other}}(X) &= \sum_{k \in J_{ft} \setminus \{j\}} 1(|d_{jkt\ell}| < \text{SD}_\ell), \\ Z_{jt\ell}^{\text{Local,Rival}}(X) &= \sum_{k \notin J_{ft}} 1(|d_{jkt\ell}| < \text{SD}_\ell), \end{aligned} \quad (5.4)$$

where  $d_{jkt\ell} = x_{kt\ell} - x_{jt\ell}$  is the difference between products  $j$  and  $k$  in terms of characteristic  $\ell$ ,  $\text{SD}_\ell$  is the standard deviation of these pairwise differences computed across all markets, and  $1(|d_{jkt\ell}| < \text{SD}_\ell)$  indicates that products  $j$  and  $k$  are close to each other in terms of characteristic  $\ell$ .

The intuition behind this “local” version is that demand for products is often most influenced by a small number of other goods that are very similar. For the “quadratic” version of  $Z^{\text{Diff}}(X)$ , which uses a more continuous measure of the distance between goods,

$$\begin{aligned} Z_{jtk}^{\text{Quad,Other}}(X) &= \sum_{k \in J_{ft} \setminus \{j\}} d_{jkt\ell}^2, \\ Z_{jtk}^{\text{Quad,Rival}}(X) &= \sum_{k \notin J_{ft}} d_{jkt\ell}^2. \end{aligned} \quad (5.5)$$

With interaction terms, which reflect covariances between different characteristics, the summands for the “local” versions are  $1(|d_{jkt\ell}| < \text{SD}_\ell) \times d_{jkt\ell'}$  for all characteristics  $\ell'$ , and the summands for the “quadratic” versions are  $d_{jkt\ell} \times d_{jkt\ell'}$  for all  $\ell' \geq \ell$ .

---

**Note:** Usually, any supply or demand shifters are added to these excluded instruments, depending on whether they are meant to be used for demand- or supply-side estimation.

---

#### Parameters

- **formulation** (*Formulation*) – *Formulation* configuration for  $X$ , the matrix of product characteristics used to build excluded instruments. Variable names should correspond to fields in `product_data`.
- **product\_data** (*structured array-like*) – Each row corresponds to a product. Markets can have differing numbers of products. The following fields are required:

- **market\_ids** : (*object*) - IDs that associate products with markets.
- **firm\_ids** : (*object*) - IDs that associate products with firms.

Along with `market_ids` and `firm_ids`, the names of any additional fields can be used as variables in `formulation`.

- **version** (*str, optional*) – The version of differentiation instruments to construct:
  - 'local' (default) - Construct the instruments in (5.4) that consider only the characteristics of “close” products in each market.
  - 'quadratic' - Construct the more continuous instruments in (5.5) that consider all products in each market.
- **interact** (*bool, optional*) – Whether to include interaction terms between different product characteristics, which can help capture covariances between product characteristics.

**Returns** Excluded differentiation instruments,  $Z^{\text{Diff}}(X)$ .

**Return type** *ndarray*

## Examples

---

The [online version](#) of the following section may be easier to read.

---

### Building Differentiation Instruments Example

```
import pyblp
import numpy as np
import pandas as pd

np.set_printoptions(precision=3)
pyblp.__version__

'0.10.1'
```

In this example, we'll load the automobile product data from *Berry, Levinsohn, and Pakes (1995)*, build some very simple excluded demand-side instruments for the problem in the spirit of *Gandhi and Houde (2017)*, and demonstrate how to update the problem data to use these instrument instead of the default ones.

```
product_data = pd.read_csv(pyblp.data.BLP_PRODUCTS_LOCATION)
product_data.head()
```

	market_ids	clustering_ids	car_ids	firm_ids	region	shares	prices	\
0	1971	AMGREM71	129	15	US	0.001051	4.935802	
1	1971	AMHORN71	130	15	US	0.000670	5.516049	
2	1971	AMJAVL71	132	15	US	0.000341	7.108642	
3	1971	AMMATA71	134	15	US	0.000522	6.839506	
4	1971	AMAMBS71	136	15	US	0.000442	8.928395	

	hpwt	air	mpd	...	supply_instruments2	supply_instruments3	\
0	0.528997	0	1.888146	...	0.0	1.705933	
1	0.494324	0	1.935989	...	0.0	1.680910	
2	0.467613	0	1.716799	...	0.0	1.801067	
3	0.426540	0	1.687871	...	0.0	1.818061	
4	0.452489	0	1.504286	...	0.0	1.933210	

	supply_instruments4	supply_instruments5	supply_instruments6	\
0	1.595656	87.0	-61.959985	
1	1.490295	87.0	-61.959985	
2	1.357703	87.0	-61.959985	
3	1.261347	87.0	-61.959985	

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```

4          1.237365          87.0          -61.959985

  supply_instruments7  supply_instruments8  supply_instruments9  \
0          0.0          46.060389          29.786989
1          0.0          46.060389          29.786989
2          0.0          46.060389          29.786989
3          0.0          46.060389          29.786989
4          0.0          46.060389          29.786989

  supply_instruments10  supply_instruments11
0          0.0          1.888146
1          0.0          1.935989
2          0.0          1.716799
3          0.0          1.687871
4          0.0          1.504286

[5 rows x 33 columns]
```

We'll first build "local" differentiation instruments, which are constructed by default, and which consist of counts of "close" rival and non-rival products in each market. Note that we're excluding the constant column because it yields collinear constant columns of differentiation instruments.

```

formulation = pyblp.Formulation('0 + hpwt + air + mpd')
local_instruments = pyblp.build_differentiation_instruments(
    formulation,
    product_data
)
local_instruments

array([[ 4.,  4.,  4., 42., 87., 83.],
       [ 4.,  4.,  4., 53., 87., 84.],
       [ 4.,  4.,  4., 51., 87., 78.],
       ...,
       [ 0.,  0.,  0., 86., 70., 62.],
       [ 1.,  1.,  1.,  3., 58., 91.],
       [ 1.,  1.,  1., 13., 58., 72.]])
```

Next, we'll build a more continuous "quadratic" version of the instruments, which consist of sums over squared differences between rival and non-rival products in each market.

```

quadratic_instruments = pyblp.build_differentiation_instruments(
    formulation,
    product_data,
    version='quadratic'
)
quadratic_instruments
array([[2.132e-02, 0.000e+00, 2.191e-01, 2.011e+00, 0.000e+00, 1.208e+01],
      [8.261e-03, 0.000e+00, 2.983e-01, 2.014e+00, 0.000e+00, 1.198e+01],
      [6.397e-03, 0.000e+00, 1.234e-01, 2.159e+00, 0.000e+00, 1.568e+01],
      ...,
      [0.000e+00, 0.000e+00, 0.000e+00, 2.239e+00, 6.000e+01, 1.312e+02],
      [1.467e-02, 0.000e+00, 6.317e-02, 1.864e+01, 7.100e+01, 6.185e+01],
      [1.467e-02, 0.000e+00, 6.317e-02, 8.961e+00, 7.100e+01, 8.819e+01]])

```

We could also use `interact=True` to include interaction terms in either version of instruments, which would help capture covariances between different product characteristics.

To use these instruments when setting up a *Problem*, the existing product data has to be updated or new product data has to be constructed. Since the existing product data is a Pandas DataFrame, it does not support matrices, so each column of instruments has to be added individually after deleting the existing instruments.

```

for i in range(8):
    del product_data[f'demand_instruments{i}']

for i, column in enumerate(local_instruments.T):
    product_data[f'demand_instruments{i}'] = column

```

product\_data

	market_ids	clustering_ids	car_ids	firm_ids	region	shares	\
0	1971	AMGREM71	129	15	US	0.001051	
1	1971	AMHORN71	130	15	US	0.000670	
2	1971	AMJAVL71	132	15	US	0.000341	
3	1971	AMMATA71	134	15	US	0.000522	
4	1971	AMAMBS71	136	15	US	0.000442	
...	...	...	...	...	...	...	...
2212	1990	VV74085	5584	6	EU	0.000488	
2213	1990	VV760G87	5585	6	EU	0.000091	
2214	1990	YGGVPL90	5589	23	EU	0.000067	
2215	1990	PS911C90	5590	12	EU	0.000039	
2216	1990	PS94490	5592	12	EU	0.000025	

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```

    prices      hpwt  air      mpd  ...  supply_instruments8  \
0      4.935802  0.528997  0  1.888146  ...      46.060389
1      5.516049  0.494324  0  1.935989  ...      46.060389
2      7.108642  0.467613  0  1.716799  ...      46.060389
3      6.839506  0.426540  0  1.687871  ...      46.060389
4      8.928395  0.452489  0  1.504286  ...      46.060389
...      ...      ...      ...      ...      ...
2212  16.140015  0.385917  1  2.639135  ...      97.039220
2213  25.986993  0.435967  1  2.136442  ...      97.039220
2214  3.393267  0.358289  0  3.518846  ...      98.024103
2215  44.758990  0.814913  1  3.016154  ...      97.222743
2216  32.058148  0.693796  1  3.267500  ...      97.222743

    supply_instruments9  supply_instruments10  supply_instruments11  \
0      29.786989      0.0      1.888146
1      29.786989      0.0      1.935989
2      29.786989      0.0      1.716799
3      29.786989      0.0      1.687871
4      29.786989      0.0      1.504286
...      ...      ...      ...
2212  27.861181      38.0      2.639135
2213  27.861181      38.0      2.136442
2214  28.809765      0.0      3.518846
2215  28.407171      19.0      3.016154
2216  28.407171      19.0      3.267500

    demand_instruments0  demand_instruments1  demand_instruments2  \
0      4.0      4.0      4.0
1      4.0      4.0      4.0
2      4.0      4.0      4.0
3      4.0      4.0      4.0
4      4.0      4.0      4.0
...      ...      ...      ...
2212  2.0      2.0      2.0
2213  2.0      2.0      2.0
2214  0.0      0.0      0.0
2215  1.0      1.0      1.0
2216  1.0      1.0      1.0

```

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	demand_instruments3	demand_instruments4	demand_instruments5
0	42.0	87.0	83.0
1	53.0	87.0	84.0
2	51.0	87.0	78.0
3	52.0	87.0	77.0
4	52.0	87.0	69.0
...	...	...	...
2212	102.0	57.0	109.0
2213	112.0	57.0	86.0
2214	86.0	70.0	62.0
2215	3.0	58.0	91.0
2216	13.0	58.0	72.0

[2217 rows x 31 columns]

Any data type that has fields can be used as product data. An alternative way to specify `problem_data` for *Problem* initialization is to simply use a `dict`, where fields can be matrices. For example, we could use the following `dict`, which includes both the new demand instruments as well as a few other variables that might be used when setting up the problem.

```
product_data_dict = {k: product_data[k] for k in ['market_ids', 'firm_ids', 'shares', 'prices', 'hpwt', 'air', 'mpd']}
product_data_dict['demand_instruments'] = local_instruments
```

## 5.2.4 pyblp.build\_id\_data

`pyblp.build_id_data(T, J, F)`

Build a balanced panel of market and firm IDs.

This function can be used to build `id_data` for *Simulation* initialization.

### Parameters

- **T** (*int*) – Number of markets.
- **J** (*int*) – Number of products in each market.
- **F** (*int*) – Number of firms. If  $J$  is divisible by  $F$ , firms produce  $J / F$  products in each market. Otherwise, firms with smaller IDs will produce excess products.

### Returns

IDs that associate products with markets and firms. Each of the  $T * J$  rows corresponds to a product. Fields:

- **market\_ids** : (*object*) - Market IDs that take on values from 0 to  $T - 1$ .
- **firm\_ids** : (*object*) - Firm IDs that take on values from 0 to  $F - 1$ .

**Return type** *recarray*

### Examples



---

The [online version](#) of the following section may be easier to read.

---

### Building ID Data Example

```
import pyblp
import numpy as np

np.set_printoptions(linewidth=1)
pyblp.__version__

'0.10.1'
```

In this example, we'll build a small panel of market and firm IDs.

```
id_data = pyblp.build_id_data(T=2, J=5, F=4)
id_data

rec.array([(0, [0]),
          (0, [0]),
          (0, [1]),
          (0, [2]),
          (0, [3]),
          (1, [0]),
          (1, [0]),
          (1, [1]),
          (1, [2]),
          (1, [3])],
          dtype=[('market_ids', 'O', (1,)), ('firm_ids', 'O', (1,))])
```

## 5.2.5 pyblp.build\_ownership

`pyblp.build_ownership` (*product\_data*, *kappa\_specification=None*)  
Build ownership matrices,  $O$ .

Ownership or product holding matrices are defined by their cooperation matrix counterparts,  $\kappa$ . For each market  $t$ ,  $\mathcal{H}_{jk} = \kappa_{fg}$  where  $j \in J_{ft}$ , the set of products produced by firm  $f$  in the market, and similarly,  $g \in J_{gt}$ .

### Parameters

- **product\_data** (*structured array-like*) – Each row corresponds to a product. Markets can have differing numbers of products. The following fields are required (except for `firm_ids` when `kappa_specification` is one of the special cases):
  - **market\_ids** : (*object*) - IDs that associate products with markets.
  - **firm\_ids** : (*object*) - IDs that associate products with firms. This field is ignored if `kappa_specification` is one of the special cases and not a function.
- **kappa\_specification** (*str or callable, optional*) – Specification for each market's cooperation matrix,  $\kappa$ , which can either be a general function or a string that implements a special case. The general function is of the following form:

```
kappa(f, g) -> value
```

where `value` is  $\mathcal{H}_{jk}$  and both `f` and `g` are firm IDs from the `firm_ids` field of `product_data`.

The default specification, `lambda: f, g: int(f == g)`, constructs traditional ownership matrices. That is,  $\kappa = I$ , the identity matrix, implies that  $\mathcal{H}_{jk}$  is 1 if the same firm produces products  $j$  and  $k$ , and is 0 otherwise.

If `firm_ids` happen to be indices for an actual  $\kappa$  matrix, `lambda f, g: kappa[f, g]` will build ownership matrices according to the matrix `kappa`.

When one of the special cases is specified, `firm_ids` in `product_data` are not required and if specified will be ignored:

- `'monopoly'` - Monopoly ownership matrices are all ones:  $\mathcal{H}_{jk} = 1$  for all  $j$  and  $k$ .
- `'single'` - Single product firm ownership matrices are identity matrices:  $\mathcal{H}_{jk} = 1$  if  $j = k$  and 0 otherwise.

**Returns** Stacked  $J_t \times J_t$  ownership matrices,  $\mathcal{H}$ , for each market  $t$ . If a market has fewer products than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

### Examples

---

The [online version](#) of the following section may be easier to read.

---

### Building Ownership Matrices Example

```
import pyblp
import numpy as np

np.set_printoptions(threshold=100)
pyblp.__version__

'0.10.1'
```

In this example, we'll use the IDs created in the *building ID data example* to build a stack of standard ownership matrices. We'll delete the first data row to demonstrate what ownership matrices should look like when markets have varying numbers of products.

```
id_data = pyblp.build_id_data(T=2, J=5, F=4)
id_data = id_data[1:]
standard_ownership = pyblp.build_ownership(id_data)
standard_ownership

array([[ 1.,  0.,  0.,  0., nan],
       [ 0.,  1.,  0.,  0., nan],
       [ 0.,  0.,  1.,  0., nan],
       [ 0.,  0.,  0.,  1., nan],
       [ 1.,  1.,  0.,  0.,  0.],
       [ 1.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  0.,  1.]])
```

We'll now modify the default  $\kappa$  specification so that the elements associated with firm IDs 0 and 1 are equal to 0.5.

```
def kappa_specification(f, g):
    if f == g:
        return 1
    return 0.5 if f < 2 and g < 2 else 0
```

We can use this specification to build a stack of alternative ownership matrices.

```
alternative_ownership = pyblp.build_ownership(id_data, kappa_specification)
alternative_ownership
```

```
array([[1. , 0.5, 0. , 0. , nan],
       [0.5, 1. , 0. , 0. , nan],
       [0. , 0. , 1. , 0. , nan],
       [0. , 0. , 0. , 1. , nan],
       [1. , 1. , 0.5, 0. , 0. ],
       [1. , 1. , 0.5, 0. , 0. ],
       [0.5, 0.5, 1. , 0. , 0. ],
       [0. , 0. , 0. , 1. , 0. ],
       [0. , 0. , 0. , 0. , 1. ]])
```

In addition to specifying a custom function, there are also a couple of special case strings that efficiently construct monopoly and single-product firm ownership matrices.

```
monopoly_ownership = pyblp.build_ownership(id_data, 'monopoly')
monopoly_ownership
```

```
array([[ 1.,  1.,  1.,  1., nan],
       [ 1.,  1.,  1.,  1., nan],
       [ 1.,  1.,  1.,  1., nan],
       [ 1.,  1.,  1.,  1., nan],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.,  1.]])
```

```
single_ownership = pyblp.build_ownership(id_data, 'single')
single_ownership
```

```
array([[ 1.,  0.,  0.,  0., nan],
       [ 0.,  1.,  0.,  0., nan],
       [ 0.,  0.,  1.,  0., nan],
       [ 0.,  0.,  0.,  1., nan],
       [ 1.,  0.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  0.,  1.]])
```

## 5.2.6 pyblp.build\_integration

`pyblp.build_integration` (*integration, dimensions*)

Build nodes and weights for integration over agent choice probabilities.

This function can be used to build custom `agent_data` for *Problem* initialization. Specifically, this function affords more flexibility than passing an *Integration* configuration directly to *Problem*. For example, if agents have unobserved tastes over only a subset of demand-side nonlinear product characteristics (i.e., if `sigma` in *Problem.solve()* has columns of zeros), this function can be used to build agent data with fewer columns of integration nodes than the number of unobserved product characteristics,  $K_2$ . This function can also be used to construct nodes that can be transformed into demographic variables.

To build nodes and weights for multiple markets, this function can be called multiple times, once for each market.

### Parameters

- **integration** (*Integration*) – *Integration* configuration for how to build nodes and weights for integration.
- **dimensions** (*int*) – Number of dimensions over which to integrate, or equivalently, the number of columns of integration nodes. When an *Integration* configuration is passed directly to *Problem*, this is the number of demand-side nonlinear product characteristics,  $K_2$ .

### Returns

Nodes and weights for integration over agent utilities. Fields:

- **weights** : (*numeric*) - Integration weights,  $w$ .
- **nodes** : (*numeric*) - Unobserved agent characteristics called integration nodes,  $\nu$ .

**Return type** *recarray*

### Examples

---

The [online version](#) of the following section may be easier to read.

---

## Building Nodes and Weights for Integration Example

```
import pyblp

pyblp.__version__

'0.10.1'
```

In this example, we'll build nodes and weights for integration over agent choice probabilities according to a *Integration* configuration. We'll construct a sparse grid of nodes and weights according to a level-5 Gauss-Hermite quadrature rule.

```
integration = pyblp.Integration('grid', 5)
integration
```

```
Configured to construct nodes and weights in a sparse grid according to the level-5 Gauss-Hermite rule with options {}.
```

Usually, this configuration should be passed directly to *Problem*, which will create a sparse grid of dimension  $K_2$ , the number of demand-side nonlinear product characteristics. Alternatively, we can build the sparse grid ourselves and pass the constructed agent data to *Problem*, possibly after modifying the nodes and weights. If we want to allow agents to have heterogeneous tastes over 2 product characteristics, we'll need a grid of dimension 2.

```
agent_data = pyblp.build_integration(integration, 2)
agent_data.nodes.shape

(53, 2)
```

```
agent_data.weights.shape

(53, 1)
```

If we wanted to construct nodes and weights for each market, we could call *build\_integration* once for each market, add a column of market IDs, and stack the arrays.

## 5.2.7 pyblp.data\_to\_dict

`pyblp.data_to_dict` (*data*, *ignore\_empty=True*)

Convert a NumPy record array into a dictionary.

Most data in PyBLP are structured as NumPy record arrays (e.g., `Problem.products` and `SimulationResults.product_data`) which can be cumbersome to work with when working with data types that can't represent matrices, such as the `pandas.DataFrame`.

This function converts record arrays created by PyBLP into dictionaries that map field names to one-dimensional arrays. Matrices in the original record array (e.g., `demand_instruments`) are split into as many fields as there are columns (e.g., `demand_instruments0`, `demand_instruments1`, and so on).

### Parameters

- **data** (*recarray*) – Record array created by PyBLP.
- **ignore\_empty** (*bool, optional*) – Whether to ignore matrices with zero size. By default, these are ignored.

**Returns** The data re-structured as a dictionary.

**Return type** *dict*

### Examples

---

The [online version](#) of the following section may be easier to read.

---

### Converting Data into a Dictionary Example

```
import pyblp
import numpy as np
import pandas as pd

np.set_printoptions(precision=1)
pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

In this example, we'll convert a dataset constructed by PyBLP into a dictionary that can more easily ingested by other Python packages.

First we'll initialize a *Problem* with the fake cereal data from *Nevo (2000)*.

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
formulation = pyblp.Formulation('0 + prices', absorb='C(product_ids)')
problem = pyblp.Problem(formulation, product_data)
problem
```

```
Dimensions:
=====
  T      N      F      K1      MD      ED
---  ---  ---  ---  ---  ---
94   2256    5    1    20    1
=====

Formulations:
=====
      Column Indices:          0
-----
X1: Linear Characteristics  prices
=====
```



The `problem.products` attribute is a typical example of the type of NumPy record array that PyBLP uses to structure data throughout the package.

```
problem.products
rec.array([(['C01Q1'], [1], ['F1B04'], [], [], ['F1B04'], [], [], [0.], [-2.5e-01, 4.1e-02, -1.6e+00, -2.7e-01, -1.0e-
↪02, 6.9e-03, -9.2e-01, 5.1e-03, 1.3e-01, 2.8e-01, 2.0e-01, 2.5e-01, -4.1e-03, -3.6e-02, 7.1e-02, 1.2e-02, 1.
↪7e-02, -1.5e-02, 8.1e-02, -1.6e-02], [], [-0.], [], [], [0.1]),
          (['C01Q1'], [1], ['F1B06'], [], [], ['F1B06'], [], [], [0.], [-2.1e-01, 5.7e-02, -1.0e+01, 1.5e-01, 4.0e-
↪02, 6.1e-03, 1.1e+00, 8.6e-02, 1.1e-01, -2.7e-02, -1.2e+00, -1.3e-01, 2.6e-03, -6.8e-03, -4.5e-02, 6.7e-05, 3.
↪1e-02, 5.8e-03, -3.2e-02, -1.1e-02], [], [-0.], [], [], [0.1]),
          (['C01Q1'], [1], ['F1B07'], [], [], ['F1B07'], [], [], [0.], [-2.1e-01, 4.6e-02, -2.3e+00, -3.0e-02, 2.4e-
↪03, -1.3e-02, 3.3e-01, -1.7e-01, -2.3e-01, 3.1e-01, 1.0e+00, 2.0e-01, 9.9e-04, 1.8e-02, 8.2e-02, 3.5e-02, 2.
↪8e-02, 1.3e-02, 4.7e-02, 2.7e-02], [], [ 0.], [], [], [0.1]),
          ...,
          (['C65Q2'], [4], ['F4B10'], [], [], ['F4B10'], [], [], [0.], [-1.2e-01, -3.2e-04, -1.1e+00, 1.8e-01, 3.6e-
↪02, -1.9e-02, 2.4e-01, 5.4e-02, -3.2e-01, 8.7e-02, 2.7e+00, 1.6e-01, 8.8e-04, 3.8e-02, 1.9e-02, -5.2e-02, -1.
↪8e-02, 3.7e-02, -5.8e-02, 3.6e-02], [], [ 0.], [], [], [0.1]),
          (['C65Q2'], [4], ['F4B12'], [], [], ['F4B12'], [], [], [0.], [-2.0e-01, 3.3e-04, -5.1e-01, -4.5e-03, 3.2e-
↪02, 6.1e-03, 5.7e-01, 2.3e-02, 1.1e-01, 1.9e-01, 2.1e+00, 1.3e-01, -8.1e-03, -1.2e-02, -3.6e-02, -4.3e-03, -1.
↪7e-02, -6.6e-03, 7.2e-03, -1.5e-02], [], [-0.], [], [], [0.1]),
          (['C65Q2'], [6], ['F6B18'], [], [], ['F6B18'], [], [], [0.], [-1.4e-01, 3.5e-03, -2.9e-01, 2.9e-01, 3.9e-
↪02, 2.0e-02, -1.9e+00, -4.0e-02, 3.8e-01, 1.1e-01, 3.4e+00, 1.1e-01, -6.1e-03, -1.2e-03, -4.7e-02, -2.4e-02, -2.
↪1e-02, -2.9e-02, -2.6e-02, -2.5e-02], [], [-0.], [], [], [0.1])),
          dtype=[('market_ids', 'O', (1,)), ('firm_ids', 'O', (1,)), ('demand_ids', 'O', (1,)), ('supply_ids', 'O', (0,
↪)), ('nesting_ids', 'O', (0,)), ('product_ids', 'O', (1,)), ('clustering_ids', 'O', (0,)), ('ownership', '<f8', (0,)),
↪ ('shares', '<f8', (1,)), ('ZD', '<f8', (20,)), ('ZS', '<f8', (0,)), ((prices,), 'X1'), '<f8', (1,)), (((), 'X2'), '
↪<f8', (0,)), (((), 'X3'), '<f8', (0,)), ('prices', '<f8', (1,))])
```

This is hard to read, and if we try to convert it into a `pandas.DataFrame`, we'll get an error. This is because `pandas.DataFrame` doesn't support matrices.

Instead, we'll use the `data_to_dict` function to first convert the record array into a dictionary, which can be easily ingested by Pandas. Matrices are converted into multiple fields, one for each column.

```
x = pyblp.data_to_dict(problem.products)
print({k: v.size for k, v in x.items()})

df = pd.DataFrame(pyblp.data_to_dict(problem.products))
df

{'market_ids': 2256, 'firm_ids': 2256, 'demand_ids': 2256, 'product_ids': 2256, 'shares': 2256, 'ZD0': 2256, 'ZD1': 2256,
↪ 'ZD2': 2256, 'ZD3': 2256, 'ZD4': 2256, 'ZD5': 2256, 'ZD6': 2256, 'ZD7': 2256, 'ZD8': 2256, 'ZD9': 2256, 'ZD10': 2256,
↪ 'ZD11': 2256, 'ZD12': 2256, 'ZD13': 2256, 'ZD14': 2256, 'ZD15': 2256, 'ZD16': 2256, 'ZD17': 2256, 'ZD18': 2256, 'ZD19': 2256, 'X1': 2256, 'prices': 2256}
```

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	market_ids	firm_ids	demand_ids	product_ids	shares	ZD0	ZD1	\
0	C01Q1	1	F1B04	F1B04	0.012417	-0.249518	0.040943	
1	C01Q1	1	F1B06	F1B06	0.007809	-0.205951	0.057100	
2	C01Q1	1	F1B07	F1B07	0.012995	-0.212031	0.046246	
3	C01Q1	1	F1B09	F1B09	0.005770	-0.170725	0.049143	
4	C01Q1	1	F1B11	F1B11	0.017934	-0.164983	0.047168	
...	...	...	...	...	...	...	...	
2251	C65Q2	3	F3B14	F3B14	0.024702	-0.126940	0.002240	
2252	C65Q2	4	F4B02	F4B02	0.007914	-0.109756	0.011192	
2253	C65Q2	4	F4B10	F4B10	0.002229	-0.119689	-0.000324	
2254	C65Q2	4	F4B12	F4B12	0.011463	-0.201890	0.000334	
2255	C65Q2	6	F6B18	F6B18	0.026208	-0.139453	0.003468	
	ZD2	ZD3	ZD4	...	ZD12	ZD13	ZD14	\
0	-1.577566	-0.269073	-0.010004	...	-0.004142	-0.035593	0.070587	
1	-10.383954	0.150476	0.039816	...	0.002585	-0.006776	-0.045453	
2	-2.278160	-0.029976	0.002390	...	0.000992	0.018425	0.081555	
3	-1.159784	-0.244789	0.002848	...	-0.004274	0.026440	0.064169	
4	-4.737563	-0.070873	0.012273	...	-0.004694	-0.029179	-0.000454	
...	...	...	...	...	...	...	...	
2251	-1.067171	0.150626	0.037091	...	-0.004787	-0.012775	-0.059399	
2252	0.458133	0.066193	0.006838	...	0.009385	0.037487	0.086225	
2253	-1.109521	0.175027	0.036227	...	0.000884	0.037634	0.019278	
2254	-0.507311	-0.004538	0.031569	...	-0.008093	-0.011750	-0.036333	
2255	-0.285143	0.291132	0.039259	...	-0.006138	-0.001181	-0.046888	
	ZD15	ZD16	ZD17	ZD18	ZD19	X1	prices	
0	0.011768	0.017287	-0.015031	0.081201	-0.015833	-0.011248	0.072088	
1	0.000067	0.031229	0.005841	-0.032121	-0.010614	-0.007135	0.114178	
2	0.034975	0.027932	0.013156	0.047484	0.026800	0.023678	0.132391	
3	0.021496	0.032372	0.033063	0.045501	0.036154	0.029725	0.130344	
4	-0.045272	-0.025446	-0.006794	-0.007560	-0.011364	-0.015585	0.154823	
...	...	...	...	...	...	...	...	
2251	0.043775	0.059339	-0.021934	0.034592	-0.021052	-0.017337	0.126086	
2252	0.060856	0.028264	0.051264	0.032965	0.033324	0.044542	0.199167	
2253	-0.052403	-0.018107	0.036733	-0.057647	0.035662	0.033720	0.137017	
2254	-0.004333	-0.017427	-0.006647	0.007228	-0.015403	-0.004174	0.100174	
2255	-0.023637	-0.021410	-0.029402	-0.025971	-0.025435	-0.011956	0.127557	

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[2256 rows x 27 columns]

## 5.3 Problem Class

Given data and appropriate configurations, a BLP-type problem can be structured by initializing the following class.

---

<code>Problem(product_formulations, product_data)</code>	A BLP-type problem.
--	---------------------

---

### 5.3.1 pyblp.Problem

```
class pyblp.Problem(product_formulations, product_data, agent_formulation=None,
                    agent_data=None, integration=None, distributions=None, epsilon_scale=1.0,
                    costs_type='linear', add_exogenous=True)
```

A BLP-type problem.

This class is initialized with relevant data and solved with `Problem.solve()`.

#### Parameters

- **product\_formulations** (*Formulation or sequence of Formulation*) – *Formulation* configuration or a sequence of up to three *Formulation* configurations for the matrix of demand-side linear product characteristics,  $X_1$ , for the matrix of demand-side nonlinear product characteristics,  $X_2$ , and for the matrix of supply-side characteristics,  $X_3$ , respectively. If the formulation for  $X_3$  is not specified or is `None`, a supply side will not be estimated. Similarly, if the formulation for  $X_2$  is not specified or is `None`, the logit (or nested logit) model will be estimated.

Variable names should correspond to fields in `product_data`. The `shares` variable should not be included in the formulations for  $X_1$  or  $X_2$ . If `shares` is included in the formulation for  $X_3$ , care should be taken when solving for equilibrium prices in, for example, `ProblemResults.compute_prices()`, since this routine assumes that marginal costs remain constant.

The `prices` variable should not be included in the formulation for  $X_3$ , but it should be included in the formulation for  $X_1$  or  $X_2$  (or both). The `absorb` argument of *Formulation* can be used to absorb fixed effects into  $X_1$  and  $X_3$ , but not  $X_2$ . Characteristics in  $X_2$  should generally be included in  $X_1$ . The typical exception is characteristics that are collinear with fixed effects that have been absorbed into  $X_1$ .

By default, characteristics in  $X_1$  that do not involve `prices`,  $X_1^{\text{ex}}$ , will be combined with excluded demand-side instruments (specified below) to create the full set of demand-side instruments,  $Z_D$ . Any fixed effects absorbed into  $X_1$  will also be absorbed into  $Z_D$ . Similarly, characteristics in  $X_3$  that do not involve `shares`,  $X_3^{\text{ex}}$ , will be combined with the excluded supply-side instruments to create  $Z_S$ , and any fixed effects absorbed into  $X_3$  will also be absorbed into  $Z_S$ . The `add_exogenous` flag can be used to disable this behavior.

**Warning:** Characteristics that involve `prices`, `p`, or `shares`, `s`, should always be formulated with the `prices` and `shares` variables, respectively. If another name is used, `Problem` will not understand that the characteristic is endogenous, so it will be erroneously included in  $Z_D$  or  $Z_S$ , and derivatives computed with respect to `prices` or `shares` will likely be wrong. For example, to include a  $p^2$  characteristic, include `I(prices**2)` in a formula instead of manually constructing and including a `prices_squared` variable.

- **product\_data** (*structured array-like*) – Each row corresponds to a product. Markets can have differing numbers of products. The following fields are required:

- **market\_ids** : (*object*) - IDs that associate products with markets.
- **shares** : (*numeric*) - Marketshares,  $s$ , which should be between zero and one, exclusive. Outside shares should also be between zero and one. Shares in each market should sum to less than one.
- **prices** : (*numeric*) - Product prices,  $p$ .

If a formulation for  $X_3$  is specified in `product_formulations`, firm IDs are also required, since they will be used to estimate the supply side of the problem:

- **firm\_ids** : (*object, optional*) - IDs that associate products with firms.

Excluded instruments are typically specified with the following fields:

- **demand\_instruments** : (*numeric*) - Excluded demand-side instruments, which, together with the formulated exogenous demand-side linear product characteristics,  $X_1^{\text{ex}}$ , constitute the full set of demand-side instruments,  $Z_D$ . To instead specify the full matrix  $Z_D$ , set `add_exogenous` to `False`.
- **supply\_instruments** : (*numeric, optional*) - Excluded supply-side instruments, which, together with the formulated exogenous supply-side characteristics,  $X_3^{\text{ex}}$ , constitute the full set of supply-side instruments,  $Z_S$ . To instead specify the full matrix  $Z_S$ , set `add_exogenous` to `False`.

The recommendation in *Conlon and Gortmaker (2020)* is to start with differentiation instruments of *Gandhi and Houde (2017)*, which can be built with `build_differentiation_instruments()`, and then compute feasible optimal instruments with `ProblemResults.compute_optimal_instruments()` in the second stage.

For guidance on how to construct instruments and add them to product data, refer to the examples in the documentation for the `build_blp_instruments()` and `build_differentiation_instruments()` functions.

If `firm_ids` are specified, custom ownership matrices can be specified as well:

- **ownership** : (*numeric, optional*) - Custom stacked  $J_t \times J_t$  ownership or product holding matrices,  $\mathcal{H}$ , for each market  $t$ , which can be built with `build_ownership()`. By default, standard ownership matrices are built only when they are needed to reduce memory usage. If specified, there should be as many columns as there are products in the market with the most products. Rightmost columns in markets with fewer products will be ignored.

---

**Note:** Fields that can have multiple columns (`demand_instruments`, `supply_instruments`, and `ownership`) can either be matrices or can be broken up into multiple one-dimensional fields with column index suffixes that start at zero. For example, if there are three columns of excluded demand-side instruments, a `demand_instruments` field with three columns can be replaced by three one-dimensional fields: `demand_instruments0`, `demand_instruments1`, and `demand_instruments2`.

---

To estimate a nested logit or random coefficients nested logit (RCNL) model, nesting groups must be specified:

- **nesting\_ids** (*object, optional*) - IDs that associate products with nesting groups. When these IDs are specified, `rho` must be specified in `Problem.solve()` as well.

To use certain types of micro moments, product IDs must be specified:

- **product\_ids** (*object, optional*) - IDs that identify individual products within markets. The IDs referenced by *DemographicExpectationMoment* or *DiversionsProbabilityMoment* must be unique within the relevant markets.

Finally, clustering groups can be specified to account for within-group correlation while updating the weighting matrix and estimating standard errors:

- **clustering\_ids** (*object, optional*) - Cluster group IDs, which will be used if `W_type` or `se_type` in `Problem.solve()` is 'clustered'.

Along with `market_ids`, `firm_ids`, `nesting_ids`, `product_ids`, `clustering_ids`, and `prices`, the names of any additional fields can typically be used as variables in `product_formulations`. However, there are a few variable names such as 'X1', which are reserved for use by *Products*.

- **agent\_formulation** (*Formulation, optional*) - *Formulation* configuration for the matrix of observed agent characteristics called demographics,  $d$ , which will only be included in the model if this formulation is specified. Since demographics are only used if there are demand-side nonlinear product characteristics, this formulation should only be specified if  $X_2$  is formulated in `product_formulations`. Variable names should correspond to fields in `agent_data`.
- **agent\_data** (*structured array-like, optional*) - Each row corresponds to an agent. Markets can have differing numbers of agents. Since simulated agents are only used if there are demand-side nonlinear product characteristics, agent data should only be specified if  $X_2$  is formulated in `product_formulations`. If agent data are specified, market IDs are required:
  - **market\_ids** : (*object*) - IDs that associate agents with markets. The set of distinct IDs should be the same as the set in `product_data`. If `integration` is specified, there must be at least as many rows in each market as the number of nodes and weights that are built for the market.

If `integration` is not specified, the following fields are required:

- **weights** : (*numeric, optional*) - Integration weights,  $w$ , for integration over agent choice probabilities.
- **nodes** : (*numeric, optional*) - Unobserved agent characteristics called integration nodes,  $v$ . If there are more than  $K_2$  columns (the number of demand-side nonlinear product characteristics), only the first  $K_2$  will be retained. If any columns of `sigma` in `Problem.solve()` are fixed at zero, only the first few columns of these nodes will be used.

The convenience function `build_integration()` can be useful when constructing custom nodes and weights.

---

**Note:** If `nodes` has multiple columns, it can be specified as a matrix or broken up into multiple one-dimensional fields with column index suffixes that start at zero. For example, if there are three columns of `nodes`, a `nodes` field with three columns can be replaced by three one-dimensional fields: `nodes0`, `nodes1`, and `nodes2`.

---

Along with `market_ids`, the names of any additional fields can be typically be used as variables in `agent_formulation`. The exception is the name 'demographics', which is reserved for use by *Agents*.

- **integration** (*Integration, optional*) - *Integration* configuration for how to build nodes and weights for integration over agent choice probabilities, which will replace any `nodes` and `weights` fields in `agent_data`. This configuration is required if `nodes`

and weights in `agent_data` are not specified. It should not be specified if  $X_2$  is not formulated in `product_formulations`.

If this configuration is specified,  $K_2$  columns of nodes (the number of demand-side nonlinear product characteristics) will be built. However, if `sigma` in `Problem.solve()` is left unspecified or specified with columns fixed at zero, fewer columns will be used.

- **distributions** (*sequence of str, optional*) – Random coefficient distributions. By default, random coefficients in (3.3) are assumed to be normally distributed. Non-default distributions can be specified with a list of the following supported strings:
  - 'normal' (default) - The random coefficient is assumed to be normal.
  - 'lognormal' - The random coefficient is assumed to be lognormal. The coefficient's column in (3.3) is exponentiated before being pre-multiplied by  $X_2$ .

The list should have as many strings as there are columns in  $X_2$ . Each string determines the distribution of the random coefficient on the corresponding product characteristic in  $X_2$ .

A typical example of a lognormal coefficient is one on prices. Implementing this typically involves having a `I(-prices)` in the formulation for  $X_2$ , and instead of including `prices` in  $X_1$ , including a `1` in the `agent_formulation`. Then the corresponding coefficient in  $\Pi$  will serve as the mean parameter for the lognormal random coefficient on negative prices,  $-p_{jt}$ .

- **epsilon\_scale** (*float, optional*) – Factor by which the Type I Extreme Value idiosyncratic preference term,  $\epsilon_{ijt}$ , is scaled. By default,  $\epsilon_{ijt}$  is not scaled. The typical use of this parameter is to approximate the pure characteristics model of [Berry and Pakes \(2007\)](#) by choosing a value smaller than `1.0`. As this scaling factor approaches zero, the model approaches the pure characteristics model in which there is no idiosyncratic preference term.

In practice, this is implemented by dividing  $V_{ijt} = \delta_{jt} + \mu_{ijt}$  by the scaling factor when solving for the mean utility  $\delta_{jt}$ . For small scaling factors, this leads to large values of  $V_{ijt}$ , which when exponentiated in the logit expression can lead to overflow issues discussed in [Berry and Pakes \(2007\)](#). The safe versions of the contraction mapping discussed in the documentation for `fp_type` in `Problem.solve()` (which is used by default) eliminate overflow issues at the cost of introducing fewer (but still common for a small scaling factor) underflow issues. Throughout the contraction mapping, some values of the simulated shares  $s_{jt}(\delta, \theta)$  can underflow to zero, causing the contraction to fail when taking logs. By default, `shares_bounds` in `Problem.solve()` bounds these simulated shares from below by `1e-300`, which eliminates these underflow issues at the cost of making it more difficult for iteration routines to converge.

With this in mind, scaling epsilon is not supported for nonlinear contractions, and is also not supported when there are nesting groups, since these further complicate the problem. In practice, if the goal is to approximate the pure characteristics model, it is a good idea to slowly decrease the scale of epsilon (e.g., starting with `0.5`, trying `0.1`, etc.) until the contraction begins to fail. To further decrease the scale, there are a few things that can help. One is passing a different `Iteration` configuration to `iteration` in `Problem.solve()`, such as `'lm'`, which can be robust in this situation. Another is to set `pyblp.options.dtype = np.longdouble` when on a system that supports extended precision (see `options` for more information about this) and choose a smaller lower bound by configuring `shares_bounds` in `Problem.solve()`. Ultimately the model will stop being solvable at a certain point, and this point will vary by problem, so approximating the pure characteristics model requires some degree of experimentation.

- **costs\_type** (*str, optional*) – Functional form of the marginal cost function  $\tilde{c} = f(c)$  in (3.9). The following specifications are supported:

- 'linear' (default) - Linear specification:  $\tilde{c} = c$ .
- 'log' - Log-linear specification:  $\tilde{c} = \log c$ .

This specification is only relevant if  $X_3$  is formulated.

- **add\_exogenous** (*bool, optional*) – Whether to add characteristics in  $X_1$  that do not involve prices,  $X_1^{\text{ex}}$ , to the `demand_instruments` field in `product_data` (including absorbed fixed effects), and similarly, whether to add characteristics in  $X_3$  that do not involve shares,  $X_3^{\text{ex}}$ , to the `supply_instruments` field. This is by default `True` so that only excluded instruments need to be specified.

If this is set to `False`, `demand_instruments` and `supply_instruments` should specify the full sets of demand- and supply-side instruments,  $Z_D$  and  $Z_S$ , and fixed effects should be manually absorbed (for example, with the `build_matrix()` function). This behavior can be useful, for example, when price is not the only endogenous product characteristic over which consumers have preferences. This model could be correctly estimated by manually adding the truly exogenous characteristics in  $X_1$  to  $Z_D$ .

**Warning:** If this flag is set to `False` because there are multiple endogenous product characteristics, care should be taken when including a supply side or computing optimal instruments. These routines assume that price is the only endogenous variable over which consumers have preferences.

### **product\_formulations**

*Formulation* configurations for  $X_1$ ,  $X_2$ , and  $X_3$ , respectively.

**Type** *Formulation or sequence of Formulation*

### **agent\_formulation**

*Formulation* configuration for  $d$ .

**Type** *Formulation*

### **products**

Product data structured as *Products*, which consists of data taken from `product_data` along with matrices built according to `Problem.product_formulations`. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *Products*

### **agents**

Agent data structured as *Agents*, which consists of data taken from `agent_data` or built by integration along with any demographics built according to `Problem.agent_formulation`. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *Agents*

### **unique\_market\_ids**

Unique market IDs in product and agent data.

**Type** *ndarray*

### **unique\_firm\_ids**

Unique firm IDs in product data.

**Type** *ndarray*

### **unique\_product\_ids**

Unique product IDs in product data.



**Type** *ndarray*

**unique\_nesting\_ids**

Unique nesting group IDs in product data.

**Type** *ndarray*

**distributions**

Random coefficient distributions.

**Type** *list of str*

**epsilon\_scale**

Factor by which the Type I Extreme Value idiosyncratic preference term,  $\epsilon_{ijt}$ , is scaled.

**Type** *float*

**costs\_type**

Functional form of the marginal cost function  $\tilde{c} = f(c)$ .

**Type** *str*

**T**

Number of markets,  $T$ .

**Type** *int*

**N**

Number of products across all markets,  $N$ .

**Type** *int*

**F**

Number of firms across all markets,  $F$ .

**Type** *int*

**I**

Number of agents across all markets,  $I$ .

**Type** *int*

**K1**

Number of demand-side linear product characteristics,  $K_1$ .

**Type** *int*

**K2**

Number of demand-side nonlinear product characteristics,  $K_2$ .

**Type** *int*

**K3**

Number of supply-side product characteristics,  $K_3$ .

**Type** *int*

**D**

Number of demographic variables,  $D$ .

**Type** *int*

**MD**

Number of demand-side instruments,  $M_D$ , which is typically the number of excluded demand-side instruments plus the number of exogenous demand-side linear product characteristics,  $K_1^{\text{ex}}$ .

**Type** *int*

**MS**

Number of supply-side instruments,  $M_S$ , which is typically the number of excluded supply-side instruments plus the number of exogenous supply-side linear product characteristics,  $K_3^{\text{ex}}$ .

**Type** *int*

**ED**

Number of absorbed dimensions of demand-side fixed effects,  $E_D$ .

**Type** *int*

**ES**

Number of absorbed dimensions of supply-side fixed effects,  $E_S$ .

**Type** *int*

**H**

Number of nesting groups,  $H$ .

**Type** *int*

**Examples**

- [Tutorial](#)

**Methods**


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<code>solve([sigma, pi, rho, beta, gamma, ...])</code>	Solve the problem.
--	--------------------

---

Once initialized, the following method solves the problem.

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<code>Problem.solve([sigma, pi, rho, beta, gamma, ...])</code>	Solve the problem.
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**5.3.2 pyblp.Problem.solve**

`Problem.solve` (*sigma=None, pi=None, rho=None, beta=None, gamma=None, sigma\_bounds=None, pi\_bounds=None, rho\_bounds=None, beta\_bounds=None, gamma\_bounds=None, delta=None, method='2s', initial\_update=False, optimization=None, scale\_objective=True, check\_optimality='both', finite\_differences=False, error\_behavior='revert', error\_punishment=1, delta\_behavior='first', iteration=None, fp\_type='safe\_linear', shares\_bounds=(1e-300, None), costs\_bounds=None, W=None, center\_moments=True, W\_type='robust', se\_type='robust', micro\_moments=(), extra\_micro\_covariances=None*)

Solve the problem.

The problem is solved in one or more GMM steps. During each step, any parameters in  $\theta$  are optimized to minimize the GMM objective value, giving the estimated  $\hat{\theta}$ . If there are no parameters in  $\theta$  (for example, in the logit model there are no nonlinear parameters and all linear parameters can be concentrated out), the objective is evaluated once during the step.

If there are nonlinear parameters, the mean utility,  $\delta(\theta)$  is computed market-by-market with fixed point iteration. Otherwise, it is computed analytically according to the solution of the logit model. If a supply side is to be estimated, marginal costs,  $c(\theta)$ , are also computed market-by-market. Linear parameters are then estimated, which are used to recover structural error terms, which in turn are used to form the objective value. By default, the objective gradient is computed as well.

---

**Note:** This method supports `parallel()` processing. If multiprocessing is used, market-by-market computation of  $\delta(\theta)$  (and  $\tilde{c}(\theta)$  if a supply side is estimated), along with associated Jacobians, will be distributed among the processes.

---

### Parameters

- **sigma** (*array-like, optional*) – Configuration for which elements in the lower-triangular Cholesky root of the covariance matrix for unobserved taste heterogeneity,  $\Sigma$ , are fixed at zero and starting values for the other elements, which, if not fixed by `sigma_bounds`, are in the vector of unknown elements,  $\theta$ .

Rows and columns correspond to columns in  $X_2$ , which is formulated according to `product_formulations` in *Problem*. If  $X_2$  was not formulated, this should not be specified, since the logit model will be estimated.

Values above the diagonal are ignored. Zeros are assumed to be zero throughout estimation and nonzeros are, if not fixed by `sigma_bounds`, starting values for unknown elements in  $\theta$ . If any columns are fixed at zero, only the first few columns of integration nodes (specified in *Problem*) will be used.

To have nonzero covariances for only a subset of the random coefficients, the characteristics for those random coefficients with zero covariances should come first in  $X_2$ . This can be seen by looking at the expression for  $\Sigma\Sigma'$ , the actual covariance matrix of the random coefficients.

- **pi** (*array-like, optional*) – Configuration for which elements in the matrix of parameters that measures how agent tastes vary with demographics,  $\Pi$ , are fixed at zero and starting values for the other elements, which, if not fixed by `pi_bounds`, are in the vector of unknown elements,  $\theta$ .

Rows correspond to the same product characteristics as in `sigma`. Columns correspond to columns in  $d$ , which is formulated according to `agent_formulation` in *Problem*. If  $d$  was not formulated, this should not be specified.

Zeros are assumed to be zero throughout estimation and nonzeros are, if not fixed by `pi_bounds`, starting values for unknown elements in  $\theta$ .

- **rho** (*array-like, optional*) – Configuration for which elements in the vector of parameters that measure within nesting group correlation,  $\rho$ , are fixed at zero and starting values for the other elements, which, if not fixed by `rho_bounds`, are in the vector of unknown elements,  $\theta$ .

If this is a scalar, it corresponds to all groups defined by the `nesting_ids` field of `product_data` in *Problem*. If this is a vector, it must have  $H$  elements, one for each nesting group. Elements correspond to group IDs in the sorted order of *Problem.unique\_nesting\_ids*. If nesting IDs were not specified, this should not be specified either.

Zeros are assumed to be zero throughout estimation and nonzeros are, if not fixed by `rho_bounds`, starting values for unknown elements in  $\theta$ .

- **beta** (*array-like, optional*) – Configuration for which elements in the vector of demand-side linear parameters,  $\beta$ , are concentrated out of the problem. Usually, this is left unspecified, unless there is a supply side, in which case parameters on endogenous product characteristics cannot be concentrated out of the problem. Values specify which elements are fixed at zero and starting values for the other elements, which, if not fixed by `beta_bounds`, are in the vector of unknown elements,  $\theta$ .

Elements correspond to columns in  $X_1$ , which is formulated according to `product_formulations` in *Problem*.

Both `None` and `numpy.nan` indicate that the parameter should be concentrated out of the problem. That is, it will be estimated, but does not have to be included in  $\theta$ . Zeros are assumed to be zero throughout estimation and nonzeros are, if not fixed by `beta_bounds`, starting values for unknown elements in  $\theta$ .

- **gamma** (*array-like, optional*) – Configuration for which elements in the vector of supply-side linear parameters,  $\gamma$ , are concentrated out of the problem. Usually, this is left unspecified. Values specify which elements are fixed at zero and starting values for the other elements, which, if not fixed by `gamma_bounds`, are in the vector of unknown elements,  $\theta$ .

Elements correspond to columns in  $X_3$ , which is formulated according to `product_formulations` in *Problem*. If  $X_3$  was not formulated, this should not be specified.

Both `None` and `numpy.nan` indicate that the parameter should be concentrated out of the problem. That is, it will be estimated, but does not have to be included in  $\theta$ . Zeros are assumed to be zero throughout estimation and nonzeros are, if not fixed by `gamma_bounds`, starting values for unknown elements in  $\theta$ .

- **sigma\_bounds** (*tuple, optional*) – Configuration for  $\Sigma$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are of the same size as `sigma`. Each element in `lb` and `ub` determines the lower and upper bound for its counterpart in `sigma`. If `optimization` does not support bounds, these will be ignored. If bounds are supported, the diagonal of `sigma` is by default bounded from below by zero.

Values above the diagonal are ignored. Lower and upper bounds corresponding to zeros in `sigma` are set to zero. Setting a lower bound equal to an upper bound fixes the corresponding element, removing it from  $\theta$ . Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **pi\_bounds** (*tuple, optional*) – Configuration for  $\Pi$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are of the same size as `pi`. Each element in `lb` and `ub` determines the lower and upper bound for its counterpart in `pi`. If `optimization` does not support bounds, these will be ignored. By default, `pi` is unbounded.

Lower and upper bounds corresponding to zeros in `pi` are set to zero. Setting a lower bound equal to an upper bound fixes the corresponding element, removing it from  $\theta$ . Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **rho\_bounds** (*tuple, optional*) – Configuration for  $\rho$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are of the same size as `rho`. Each element in `lb` and `ub` determines the lower and upper bound for its counterpart in `rho`. If `optimization` does not support bounds, these will be ignored.

If bounds are supported, `rho` is by default bounded from below by 0, which corresponds to the simple logit model, and bounded from above by 0.99 because values greater than 1 are inconsistent with utility maximization.

Lower and upper bounds corresponding to zeros in `rho` are set to zero. Setting a lower bound equal to an upper bound fixes the corresponding element, removing it from  $\theta$ . Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **beta\_bounds** (*tuple, optional*) – Configuration for  $\beta$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are of the same size as `beta`. Each element in `lb` and `ub` determines the lower and upper bound for its counterpart in `beta`. If `optimization` does not support bounds, these will be ignored.

Usually, this is left unspecified unless there is a supply side, in which case parameters on endogenous product characteristics cannot be concentrated out of the problem. It is generally a good idea to constrain such parameters to be nonzero so that the intra-firm Jacobian of shares with respect to prices does not become singular.

By default, all non-concentrated out parameters are unbounded. Bounds should only be specified for parameters that are included in  $\theta$ ; that is, those with initial values specified in `beta`.

Lower and upper bounds corresponding to zeros in `beta` are set to zero. Setting a lower bound equal to an upper bound fixes the corresponding element, removing it from  $\theta$ . Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **`gamma_bounds`** (*tuple, optional*) – Configuration for  $\gamma$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are of the same size as `gamma`. Each element in `lb` and `ub` determines the lower and upper bound for its counterpart in `gamma`. If `optimization` does not support bounds, these will be ignored.

By default, all non-concentrated out parameters are unbounded. Bounds should only be specified for parameters that are included in  $\theta$ ; that is, those with initial values specified in `gamma`.

Lower and upper bounds corresponding to zeros in `gamma` are set to zero. Setting a lower bound equal to an upper bound fixes the corresponding element, removing it from  $\theta$ . Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **`delta`** (*array-like, optional*) – Initial values for the mean utility,  $\delta$ . If there are any nonlinear parameters, these are the values at which the fixed point iteration routine will start during the first objective evaluation. By default, the solution to the logit model in (3.40) is used. If  $\rho$  is specified, the solution to the nested logit model in (3.41) under the initial `rho` is used instead.
- **`method`** (*str, optional*) – The estimation routine that will be used. The following methods are supported:
  - `'1s'` - One-step GMM.
  - `'2s'` (default) - Two-step GMM.

Iterated GMM can be manually implemented by executing single GMM steps in a loop, in which after the first iteration, nonlinear parameters and weighting matrices from the last `ProblemResults` are passed as arguments.

- **`initial_update`** (*bool, optional*) – Whether to update starting values for the mean utility  $\delta$ , and the weighting matrix,  $W$ , at the initial parameter values before the first GMM step (this initial update will be called a zeroth step). By default, initial values are not updated because this requires an additional objective evaluation.
- **`optimization`** (*Optimization, optional*) – `Optimization` configuration for how to solve the optimization problem in each GMM step, which is only used if there are unfixed nonlinear parameters over which to optimize. By default, `Optimization('l-bfgs-b', {'ftol': 0, 'gtol': 1e-8})` is used. If available, `Optimization('knitro')` may be preferable. Generally, it is recommended to consider a number of different optimization routines and starting values, verifying that  $\hat{\theta}$  satisfies both the first and second order conditions. Choosing a routine that supports bounds (and configuring bounds) is typically a good idea. Choosing a routine that does not use analytic gradients will often down estimation.
- **`scale_objective`** (*bool, optional*) – Whether to scale the objective in (3.10) by  $N$ , the number of observations, in which case the objective after two GMM steps is equal to the  $J$

statistic from *Hansen (1982)*. By default, the objective is scaled by  $N$ .

In theory the scale of the objective should not matter, but in practice having similar objective values for different problem sizes is helpful because similar optimization tolerances can be used.

- **check\_optimality** (*str, optional*) – How to check for optimality (first and second order conditions) after the optimization routine finishes. The following configurations are supported:
  - 'gradient' - Analytically compute the gradient after optimization finishes, but do not compute the Hessian. Since Jacobians needed to compute standard errors will already be computed, gradient computation will not take a long time. This option may be useful if Hessian computation takes a long time when, for example, there are a large number of parameters.
  - 'both' (default) - Also compute the Hessian with central finite differences after optimization finishes.
- **finite\_differences** (*bool, optional*) – Whether to use finite differences to compute Jacobians and the gradient instead of analytic expressions. Since finite differences comes with numerical approximation error and is typically slower, analytic expressions are used by default.

One situation in which finite differences may be preferable is when there are a sufficiently large number of products and integration nodes in individual markets so as to make computing analytic Jacobians infeasible because of large memory requirements. Note that an analytic expression for the Hessian has not been implemented, so when computed it is always approximated with finite differences.

- **error\_behavior** (*str, optional*) – How to handle any errors. For example, there can sometimes be overflow or underflow when computing  $\delta(\theta)$  at a large  $\hat{\theta}$ . The following behaviors are supported:
  - 'revert' (default) - Revert problematic values to their last computed values. If there are problematic values during the first objective evaluation, revert values in  $\delta(\theta)$  to their starting values; in  $\tilde{c}(\hat{\theta})$ , to prices; in the objective, to  $1e10$ ; and in other matrices such as Jacobians, to zeros.
  - 'punish' - Set the objective to 1 and its gradient to all zeros. This option along with a large `error_punishment` can be helpful for routines that do not use analytic gradients.
  - 'raise' - Raise an exception.
- **error\_punishment** (*float, optional*) – How to scale the GMM objective value after an error. By default, the objective value is not scaled.
- **delta\_behavior** (*str, optional*) – Configuration for the values at which the fixed point computation of  $\delta(\theta)$  in each market will start. This configuration is only relevant if there are unfixed nonlinear parameters over which to optimize. The following behaviors are supported:
  - 'first' (default) - Start at the values configured by `delta` during the first GMM step, and at the values computed by the last GMM step for each subsequent step.
  - 'last' - Start at the values of  $\delta(\theta)$  computed during the last objective evaluation, or, if this is the first evaluation, at the values configured by `delta`. This behavior tends to speed up computation but may introduce some instability into estimation.

- **iteration** (*Iteration, optional*) – *Iteration* configuration for how to solve the fixed point problem used to compute  $\delta(\theta)$  in each market. This configuration is only relevant if there are nonlinear parameters, since  $\delta$  can be estimated analytically in the logit model. By default, `Iteration('squarem', {'atol': 1e-14})` is used. Newton-based routines such as `Iteration('lm')` that compute the Jacobian can often be faster (especially when there are nesting parameters), but the Jacobian-free SQUAREM routine is used by default because its speed is often comparable and in practice it can be slightly more stable.
- **fp\_type** (*str, optional*) – Configuration for the type of contraction mapping used to compute  $\delta(\theta)$ . The following types are supported:
  - `'safe_linear'` (default) - The standard linear contraction mapping in (3.13) (or (3.39) when there is nesting) with safeguards against numerical overflow. Specifically,  $\max_j V_{ijt}$  (or  $\max_j V_{ijt}/(1 - \rho_{h(j)})$  when there is nesting) is subtracted from  $V_{ijt}$  and the logit expression for choice probabilities in (3.5) (or (3.37)) is re-scaled accordingly. Such re-scaling is known as the log-sum-exp trick.
  - `'linear'` - The standard linear contraction mapping without safeguards against numerical overflow. This option may be preferable to `'safe_linear'` if utilities are reasonably small and unlikely to create overflow problems.
  - `'nonlinear'` - Iteration over  $\exp \delta_{jt}$  instead of  $\delta_{jt}$ . This can be faster than `'linear'` because it involves fewer logarithms. Also, following *Brunner, Heiss, Romahn, and Weiser (2017)*, the  $\exp \delta_{jt}$  term can be cancelled out of the expression because it also appears in the numerator of (3.5) in the definition of  $s_{jt}(\delta, \theta)$ . This second trick only works when there are no nesting parameters.
  - `'safe_nonlinear'` - Exponentiated version with minimal safeguards against numerical overflow. Specifically,  $\max_j \mu_{ijt}$  is subtracted from  $\mu_{ijt}$ . This helps with stability but is less helpful than subtracting from the full  $V_{ijt}$ , so this version is less stable than `'safe_linear'`.

This option is only relevant if `sigma` or `pi` are specified because  $\delta$  can be estimated analytically in the logit model with (3.40) and in the nested logit model with (3.41).

- **shares\_bounds** (*tuple, optional*) – Configuration for  $s_{jt}(\delta, \theta)$  bounds in the contraction in (3.13) of the form `(lb, ub)`, in which both `lb` and `ub` are floats or `None`. By default, simulated shares are bounded from below by `1e-300`. This is only relevant if `fp_type` is `'safe_linear'` or `'linear'`. Bounding shares in the contraction does nothing with a nonlinear fixed point.

It can be particularly helpful to bound shares in the contraction from below by a small number to prevent the contraction from failing when there are issues with zero or negative simulated shares. Zero shares can occur when there are underflow issues and negative shares can occur when there are issues with the numerical integration routine having negative integration weights (e.g., for sparse grid integration).

The idea is that a small lower bound will allow the contraction to converge even when it encounters some issues with small or negative shares. However, if these issues are unlikely, disabling this behavior can speed up the iteration routine because fewer checks will be done.

Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **costs\_bounds** (*tuple, optional*) – Configuration for  $c_{jt}(\theta)$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are floats or `None`. This is only relevant if  $X_3$  was formulated by `product_formulations` in *Problem*. By default, marginal costs are unbounded.

When `costs_type` in `Problem` is 'log', nonpositive  $c(\theta)$  values can create problems when computing  $\tilde{c}(\theta) = \log c(\theta)$ . One solution is to set `lb` to a small number. Rows in Jacobians associated with clipped marginal costs will be zero.

Both `None` and `numpy.nan` are converted to `-numpy.inf` in `lb` and to `numpy.inf` in `ub`.

- **`W`** (*array-like, optional*) – Starting values for the weighting matrix,  $W$ . By default, the 2SLS weighting matrix in (3.23) is used.

If there are any `micro_moments`, the initial weighting matrix will by default be block-diagonal with an identity matrix for the micro moment block. This micro moment block should usually be replaced by a matrix that better reflects micro moment covariances and the size of the micro dataset relative to  $N$ .

- **`center_moments`** (*bool, optional*) – Whether to center each column of the demand- and supply-side moments  $g$  before updating the weighting matrix  $W$  according to (3.24). By default, the moments are centered. This has no effect if `W_type` is 'unadjusted'.
- **`W_type`** (*str, optional*) – How to update the weighting matrix. This has no effect if `method` is 'ls'. Usually, `se_type` should be the same. The following types are supported:
  - 'robust' (default) - Heteroscedasticity robust weighting matrix defined in (3.24) and (3.25).
  - 'clustered' - Clustered weighting matrix defined in (3.24) and (3.26). Clusters must be defined by the `clustering_ids` field of `product_data` in `Problem`.
  - 'unadjusted' - Homoskedastic weighting matrix defined in (3.24) and (3.28).

This only affects the standard demand- and supply-side block of the updated weighting matrix. If there are micro moments, this matrix will be block-diagonal with a micro moment block equal to the inverse of the covariance matrix defined in (3.35) plus any `extra_micro_covariances`.

- **`se_type`** (*str, optional*) – How to compute parameter covariances and standard errors. Usually, `W_type` should be the same. The following types are supported:
  - 'robust' (default) - Heteroscedasticity robust covariances defined in (3.30) and (3.25).
  - 'clustered' - Clustered covariances defined in (3.30) and (3.26). Clusters must be defined by the `clustering_ids` field of `product_data` in `Problem`.
  - 'unadjusted' - Homoskedastic covariances defined in (3.31), which are computed under the assumption that the weighting matrix is optimal.

This only affects the standard demand- and supply-side block of the matrix of averaged moment covariances. If there are micro moments, the  $S$  matrix defined in the expressions referenced above will be block-diagonal with a micro moment block equal to the covariance matrix defined in (3.35) plus any `extra_micro_covariances`.

- **`micro_moments`** (*sequence of Moment, optional*) – Configurations for the  $M_M$  micro moments that will be added to the standard set of moments. For a list of supported micro moments, refer to *Micro Moment Classes*. By default, no micro moments are used, so  $M_M = 0$ .

If micro moments are specified, the micro moment block in `W` should usually be replaced by a matrix that better reflects micro moment covariances and the size of the micro dataset relative to  $N$ . If micro moments were computed with substantial sampling error, `extra_micro_covariances` can be specified to account for this additional source of error.



- **extra\_micro\_covariances** (*array-like, optional*) – Covariance matrix that is added on to the  $M_M \times M_M$  matrix of micro moments covariances defined in (3.35), which is used to update the weighting matrix and compute standard errors. By default, this matrix is assumed to be zero. It should be specified if, for example, micro moments were computed with substantial sampling error.

**Returns** *ProblemResults* of the solved problem.

**Return type** *ProblemResults*

### Examples

- *Tutorial*

## 5.4 Micro Moment Classes

Micro moment configurations can be passed to *Problem.solve()*.

<i>DemographicExpectationMoment</i> (product_id, ...)	Configuration for micro moments that match expectations of demographics for agents who choose certain products.
<i>DemographicCovarianceMoment</i> (X2_index, ..., ...)	Configuration for micro moments that match covariances between product characteristics and demographics.
<i>DiversioProbabilityMoment</i> (product_id1, ...)	Configuration for micro moments that match second choice probabilities of certain products for agents whose first choices are certain other products.
<i>DiversioCovarianceMoment</i> (X2_index1, ..., ...)	Configuration for micro moments that match covariances between product characteristics of first and second choices.

### 5.4.1 pyblp.DemographicExpectationMoment

**class** `pyblp.DemographicExpectationMoment` (*product\_id, demographics\_index, values, market\_ids=None*)

Configuration for micro moments that match expectations of demographics for agents who choose certain products.

For example, micro data can sometimes be used to compute the mean of a demographic such as income,  $y_{it}$ , for agents who choose product  $j$ . With the value  $\mathcal{V}_{mt}$  of this mean, a micro moment  $m$  in market  $t$  can be defined by  $g_{M,mt} = \mathcal{V}_{mt} - v_{mt}$  where

$$v_{mt} = \frac{E[y_{it}s_{ijt}]}{s_{jt}}. \quad (5.6)$$

Integrals of these micro moments are averaged across a set  $T_m$  of markets, which gives  $\bar{g}_{M,m}$  in (3.34).

#### Parameters

- **product\_id** (*object*) – ID of the product  $j$  or `None` to denote the outside option  $j = 0$ . If not `None`, there must be exactly one of this ID in the `product_ids` field of `product_data` in *Problem* or *Simulation* for each market over which this micro moment will be averaged.

- **demographics\_index** (*int*) – Column index of the demographic  $y_{it}$  (which can be any demographic, not just income) in the matrix of agent demographics,  $d$ . This should be between zero and  $D - 1$ , inclusive.
- **values** (*float*) – Values  $\mathcal{V}_{mt}$  of the statistic estimated from micro data. If a scalar is specified, then  $\mathcal{V}_{mt} = \mathcal{V}_m$  is assumed to be constant across all markets in which the moment is relevant. Otherwise, this should have as many elements as `market_ids`, or as the total number of markets if `market_ids` is `None`.
- **market\_ids** (*array-like, optional*) – Distinct market IDs over which the micro moments will be averaged to get  $\bar{g}_{M,m}$ . These are also the only markets in which the moments will be computed. By default, the moments are computed for and averaged across all markets.

## Examples

- *Tutorial*

## Methods

### 5.4.2 pyblp.DemographicCovarianceMoment

**class** `pyblp.DemographicCovarianceMoment` (*X2\_index*, *demographics\_index*, *values*, *market\_ids=None*)

Configuration for micro moments that match covariances between product characteristics and demographics.

For example, micro data can sometimes be used to compute the sample covariance between a product characteristic  $x_{jt}$  of an agent's choice  $j$ , and a demographic such as income,  $y_{it}$ , amongst those agents who purchase an inside good. With the value  $\mathcal{V}_{mt}$  of this sample covariance, a micro moment  $m$  in market  $t$  can be defined by  $g_{M,mt} = \mathcal{V}_{mt} - v_{mt}$  where

$$v_{mt} = \text{Cov}(y_{it}, z_{it}) \quad (5.7)$$

where conditional on choosing an inside good, the expected value of  $x_{jt}$  for agent  $i$  is

$$z_{it} = \sum_{j \in J_t} x_{jt} s_{ij(-0)t} \quad (5.8)$$

where  $s_{ij(-0)t} = s_{ijt}/(1 - s_{i0t})$  is the probability of  $i$  choosing  $j$  when the outside option is removed from the choice set.

Integrals of these micro moments are averaged across a set  $T_m$  of markets, which gives  $\bar{g}_{M,m}$  in (3.34).

#### Parameters

- **X2\_index** (*int*) – Column index of  $x_{jt}$  in the matrix of demand-side nonlinear product characteristics,  $X_2$ . This should be between zero and  $K_2 - 1$ , inclusive.
- **demographics\_index** (*int*) – Column index of the demographic  $y_{it}$  (which can be any demographic, not just income) in the matrix of agent demographics,  $d$ . This should be between zero and  $D - 1$ , inclusive.
- **values** (*float*) – Values  $\mathcal{V}_{mt}$  of the statistic estimated from micro data. If a scalar is specified, then  $\mathcal{V}_{mt} = \mathcal{V}_m$  is assumed to be constant across all markets in which the moment is relevant. Otherwise, this should have as many elements as `market_ids`, or as the total number of markets if `market_ids` is `None`.

- **market\_ids** (*array-like, optional*) – Distinct market IDs over which the micro moments will be averaged to get  $\bar{g}_{M,m}$ . These are also the only markets in which the moments will be computed. By default, the moments are computed for and averaged across all markets.

## Examples

- *Tutorial*

## Methods

### 5.4.3 pyblp.DiversionProbabilityMoment

**class** `pyblp.DiversionProbabilityMoment` (*product\_id1*, *product\_id2*, *values*, *market\_ids=None*)

Configuration for micro moments that match second choice probabilities of certain products for agents whose first choices are certain other products.

For example, micro data can sometimes be used to compute the share of agents who would choose product  $k$  if  $j$  were removed from the choice set, out of those agents whose first choice is  $j$ . With the value  $\mathcal{V}_{mt}$  of this share, a micro moment  $m$  in market  $t$  can be defined by  $g_{M,mt} = \mathcal{V}_{mt} - v_{mt}$  where

$$v_{mt} = \frac{E[s_{ik(-j)t}s_{ijt}]}{s_{jt}} \quad (5.9)$$

where  $s_{ik(-j)t} = s_{ijt}/(1 - s_{ijt})$  is the probability of  $i$  choosing  $k$  when  $j$  is removed from the choice set. Rearranging terms gives the equivalent definition

$$g_{M,mt} = \mathcal{V}_{mt} - \frac{s_{k(-j)t} - s_{kt}}{s_{jt}}, \quad (5.10)$$

which is more reminiscent of the long-run diversion ratios  $\bar{\mathcal{D}}_{jk}$  computed by `ProblemResults.compute_long_run_diversion_ratios()`.

Integrals of these micro moments are averaged across a set  $T_m$  of markets, which gives  $\bar{g}_{M,m}$  in (3.34).

#### Parameters

- **product\_id1** (*object*) – ID of the first choice product  $j$  or `None` to denote the outside option  $j = 0$ . There must be exactly one of this ID in the `product_ids` field of `product_data` in `Problem` or `Simulation` for each market over which this micro moment will be averaged.
- **product\_id2** (*object*) – ID of the second choice product  $k$  or `None` to denote the outside option  $j = 0$ . If not `None`, there must be exactly one of this ID for each market over which this micro moment will be averaged.
- **values** (*float*) – Values  $\mathcal{V}_{mt}$  of the statistic estimated from micro data. If a scalar is specified, then  $\mathcal{V}_{mt} = \mathcal{V}_m$  is assumed to be constant across all markets in which the moment is relevant. Otherwise, this should have as many elements as `market_ids`, or as the total number of markets if `market_ids` is `None`.
- **market\_ids** (*array-like, optional*) – Distinct market IDs over which the micro moments will be averaged to get  $\bar{g}_{M,m}$ . These are also the only markets in which the moments will be computed. By default, the moments are computed for and averaged across all markets.

## Examples

- [Tutorial](#)

## Methods

### 5.4.4 pyblp.DiversionCovarianceMoment

**class** `pyblp.DiversionCovarianceMoment` (*X2\_index1*, *X2\_index2*, *values*, *market\_ids=None*)

Configuration for micro moments that match covariances between product characteristics of first and second choices.

For example, survey data can sometimes be used to compute the sample covariance between a product characteristic  $x_{jt}^{(1)}$  of an agent's first choice  $j$  and either the same or a different product characteristic  $x_{kt}^{(2)}$  of the agent's second choice  $k$  if  $j$  were removed from the choice set, amongst those agents whose first and second choices are both inside goods. With the value  $\mathcal{V}_{mt}$  of this sample covariance, a micro moment  $m$  in market  $t$  can be defined by  $g_{M,mt} = \mathcal{V}_{mt} - v_{mt}$  where

$$v_{mt} = \text{Cov}(z_{it}^{(1)}, z_{it}^{(2)}) \quad (5.11)$$

where conditional on purchasing inside goods, the expected values of  $x_{jt}^{(1)}$  and  $x_{kt}^{(2)}$  for agent  $i$  are

$$z_{it}^{(1)} = \sum_{j \in J_t} x_{jt}^{(1)} s_{ij(-0)t}, \quad z_{it}^{(2)} = \sum_{j,k \in J_t} x_{kt}^{(2)} s_{ik(-0,j)t} s_{ij(-0)t} \quad (5.12)$$

where  $s_{ij(-0)t}$  is the probability of choosing  $j$  when the outside option is removed from the choice set and  $s_{ik(-0,j)t}$  is the probability of choosing  $k$  when both the outside option and  $j$  are removed from the choice set.

Integrals of these micro moments are averaged across a set  $T_m$  of markets, which gives  $\bar{g}_{M,m}$  in (3.34).

#### Parameters

- **X2\_index1** (*int*) – Column index of  $x_{jt}^{(1)}$  in the matrix of demand-side nonlinear product characteristics,  $X_2$ . This should be between zero and  $K_2 - 1$ , inclusive.
- **X2\_index2** (*int*) – Column index of  $x_{kt}^{(2)}$  in the matrix of demand-side nonlinear product characteristics,  $X_2$ . This should be between zero and  $K_2 - 1$ , inclusive.
- **values** (*float*) – Values  $\mathcal{V}_{mt}$  of the statistic estimated from micro data. If a scalar is specified, then  $\mathcal{V}_{mt} = \mathcal{V}_m$  is assumed to be constant across all markets in which the moment is relevant. Otherwise, this should have as many elements as `market_ids`, or as the total number of markets if `market_ids` is `None`.
- **market\_ids** (*array-like, optional*) – Distinct market IDs over which the micro moments will be averaged to get  $\bar{g}_{M,m}$ . These are also the only markets in which the moments will be computed. By default, the moments are computed for and averaged across all markets.

## Examples

- [Tutorial](#)

## Methods

## 5.5 Problem Results Class

Solved problems return the following results class.

---

<i>ProblemResults</i>	Results of a solved BLP problem.
-----------------------	----------------------------------

---

### 5.5.1 pyblp.ProblemResults

**class** `pyblp.ProblemResults`

Results of a solved BLP problem.

Many results are class attributes. Other post-estimation outputs be computed by calling class methods.

---

**Note:** Methods in this class that compute one or more post-estimation output per market support `parallel()` processing. If multiprocessing is used, market-by-market computation of each post-estimation output will be distributed among the processes.

---

**problem**

*Problem* that created these results.

**Type** *Problem*

**last\_results**

*ProblemResults* from the last GMM step.

**Type** *ProblemResults*

**step**

GMM step that created these results.

**Type** *int*

**optimization\_time**

Number of seconds it took the optimization routine to finish.

**Type** *float*

**cumulative\_optimization\_time**

Sum of *ProblemResults.optimization\_time* for this step and all prior steps.

**Type** *float*

**total\_time**

Sum of *ProblemResults.optimization\_time* and the number of seconds it took to set up the GMM step and compute results after optimization had finished.

**Type** *float*

**cumulative\_total\_time**

Sum of *ProblemResults.total\_time* for this step and all prior steps.

**Type** *float*

**converged**

Whether the optimization routine converged.

Type *bool*

**cumulative\_converged**

Whether the optimization routine converged for this step and all prior steps.

Type *bool*

**optimization\_iterations**

Number of major iterations completed by the optimization routine.

Type *int*

**cumulative\_optimization\_iterations**

Sum of *ProblemResults.optimization\_iterations* for this step and all prior steps.

Type *int*

**objective\_evaluations**

Number of GMM objective evaluations.

Type *int*

**cumulative\_objective\_evaluations**

Sum of *ProblemResults.objective\_evaluations* for this step and all prior steps.

Type *int*

**fp\_converged**

Flags for convergence of the iteration routine used to compute  $\delta(\theta)$  in each market during each objective evaluation. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to objective evaluations.

Type *ndarray*

**cumulative\_fp\_converged**

Concatenation of *ProblemResults.fp\_converged* for this step and all prior steps.

Type *ndarray*

**fp\_iterations**

Number of major iterations completed by the iteration routine used to compute  $\delta(\theta)$  in each market during each objective evaluation. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to objective evaluations.

Type *ndarray*

**cumulative\_fp\_iterations**

Concatenation of *ProblemResults.fp\_iterations* for this step and all prior steps.

Type *ndarray*

**contraction\_evaluations**

Number of times the contraction used to compute  $\delta(\theta)$  was evaluated in each market during each objective evaluation. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to objective evaluations.

Type *ndarray*

**cumulative\_contraction\_evaluations**

Concatenation of *ProblemResults.contraction\_evaluations* for this step and all prior steps.

Type *ndarray*

**parameters**

Stacked parameters in the following order:  $\hat{\theta}$ , concentrated out elements of  $\hat{\beta}$ , and concentrated out elements of  $\hat{\gamma}$ .

Type *ndarray*

**parameter\_covariances**

Estimated covariance matrix of the stacked parameters, from which standard errors are extracted. Parameter covariances are not estimated during the first step of two-step GMM.

Type *ndarray*

**theta**

Estimated unfixed parameters,  $\hat{\theta}$ , in the following order:  $\hat{\Sigma}$ ,  $\hat{\Pi}$ ,  $\hat{\rho}$ , non-concentrated out elements from  $\hat{\beta}$ , and non-concentrated out elements from  $\hat{\gamma}$ .

Type *ndarray*

**sigma**

Estimated Cholesky root of the covariance matrix for unobserved taste heterogeneity,  $\hat{\Sigma}$ .

Type *ndarray*

**sigma\_squared**

Estimated covariance matrix for unobserved taste heterogeneity,  $\hat{\Sigma}\hat{\Sigma}'$ .

Type *ndarray*

**pi**

Estimated parameters that measures how agent tastes vary with demographics,  $\hat{\Pi}$ .

Type *ndarray*

**rho**

Estimated parameters that measure within nesting group correlations,  $\hat{\rho}$ .

Type *ndarray*

**beta**

Estimated demand-side linear parameters,  $\hat{\beta}$ .

Type *ndarray*

**gamma**

Estimated supply-side linear parameters,  $\hat{\gamma}$ .

Type *ndarray*

**sigma\_se**

Estimated standard errors for  $\hat{\Sigma}$ , which are not estimated in the first step of two-step GMM.

Type *ndarray*

**sigma\_squared\_se**

Estimated standard errors for  $\hat{\Sigma}\hat{\Sigma}'$ , which are computed with the delta method, and are not estimated in the first step of two-step GMM.

Type *ndarray*

**pi\_se**

Estimated standard errors for  $\hat{\Pi}$ , which are not estimated in the first step of two-step GMM.

Type *ndarray*

**rho\_se**

Estimated standard errors for  $\hat{\rho}$ , which are not estimated in the first step of two-step GMM.

**Type** *ndarray*

**beta\_se**

Estimated standard errors for  $\hat{\beta}$ , which are not estimated in the first step of two-step GMM.

**Type** *ndarray*

**gamma\_se**

Estimated standard errors for  $\hat{\gamma}$ , which are not estimated in the first step of two-step GMM.

**Type** *ndarray*

**sigma\_bounds**

Bounds for  $\Sigma$  that were used during optimization, which are of the form (lb, ub).

**Type** *tuple*

**pi\_bounds**

Bounds for  $\Pi$  that were used during optimization, which are of the form (lb, ub).

**Type** *tuple*

**rho\_bounds**

Bounds for  $\rho$  that were used during optimization, which are of the form (lb, ub).

**Type** *tuple*

**beta\_bounds**

Bounds for  $\beta$  that were used during optimization, which are of the form (lb, ub).

**Type** *tuple*

**gamma\_bounds**

Bounds for  $\gamma$  that were used during optimization, which are of the form (lb, ub).

**Type** *tuple*

**sigma\_labels**

Variable labels for rows and columns of  $\Sigma$ , which are derived from the formulation for  $X_2$ .

**Type** *list of str*

**pi\_labels**

Variable labels for columns of  $\Pi$ , which are derived from the formulation for demographics.

**Type** *list of str*

**rho\_labels**

Variable labels for  $\rho$ . If  $\rho$  is not a scalar, this is *Problem.unique\_nesting\_ids*.

**Type** *list of str*

**beta\_labels**

Variable labels for  $\beta$ , which are derived from the formulation for  $X_1$ .

**Type** *list of str*

**gamma\_labels**

Variable labels for  $\gamma$ , which are derived from the formulation for  $X_3$ .

**Type** *list of str*

**delta**

Estimated mean utility,  $\delta(\hat{\theta})$ .

**Type** *ndarray*



**clipped\_shares**

Vector of booleans indicator whether the associated simulated shares were clipped during the last fixed point iteration to compute  $\delta(\hat{\theta})$ . All elements will be `False` if `shares_bounds` in `Problem.solve()` is disabled (by default shares are bounded from below by a small number to alleviate issues with underflow and negative shares).

Type `ndarray`

**tilde\_costs**

Estimated transformed marginal costs,  $\tilde{c}(\hat{\theta})$  from (3.9). If `costs_bounds` were specified in `Problem.solve()`, `c` may have been clipped.

Type `ndarray`

**clipped\_costs**

Vector of booleans indicating whether the associated marginal costs were clipped. All elements will be `False` if `costs_bounds` in `Problem.solve()` was not specified.

Type `ndarray`

**xi**

Estimated unobserved demand-side product characteristics,  $\xi(\hat{\theta})$ , or equivalently, the demand-side structural error term. When there are demand-side fixed effects, this is  $\Delta\xi(\hat{\theta})$  in (3.32). That is, fixed effects are not included.

Type `ndarray`

**omega**

Estimated unobserved supply-side product characteristics,  $\omega(\hat{\theta})$ , or equivalently, the supply-side structural error term. When there are supply-side fixed effects, this is  $\Delta\omega(\hat{\theta})$  in (3.32). That is, fixed effects are not included.

Type `ndarray`

**micro**

Averaged micro moments,  $\bar{g}_M$ , in (3.34).

Type `ndarray`

**micro\_values**

Simulated micro moment values,  $v_{mt}$ . Rows are in the same order as `Problem.unique_market_ids`. Columns are in the same order as `ProblemResults.micro`. If a micro moment is not computed in one or more markets, the associated values will be `numpy.nan`.

Type `ndarray`

**objective**

GMM objective value,  $q(\hat{\theta})$ , defined in (3.10). If `scale_objective` was `True` in `Problem.solve()` (which is the default), this value was scaled by  $N$  so that objective values are more comparable across different problem sizes. Note that in some of the BLP literature (and earlier versions of this package), this expression was previously scaled by  $N^2$ .

Type `float`

**xi\_by\_theta\_jacobian**

Estimated  $\frac{\partial \xi}{\partial \theta} = \frac{\partial \delta}{\partial \theta}$ , which is used to compute the gradient and standard errors.

Type `ndarray`

**omega\_by\_theta\_jacobian**

Estimated  $\frac{\partial \omega}{\partial \theta} = \frac{\partial \tilde{c}}{\partial \theta}$ , which is used to compute the gradient and standard errors.

Type `ndarray`

**micro\_by\_theta\_jacobian**

Estimated  $\frac{\partial \bar{q}_M}{\partial \theta}$ , which is used to compute the gradient and standard errors.

Type *ndarray*

**gradient**

Gradient of the GMM objective,  $\nabla q(\hat{\theta})$ , defined in (3.18). This is computed after the optimization routine finishes even if the routine was configured to not use analytic gradients.

Type *ndarray*

**projected\_gradient**

Projected gradient of the GMM objective. When there are no parameter bounds, this will always be equal to *ProblemResults.gradient*. Otherwise, if an element in  $\hat{\theta}$  is equal to its lower (upper) bound, the corresponding projected gradient value will be truncated at a maximum (minimum) of zero.

Type *ndarray*

**projected\_gradient\_norm**

Infinity norm of *ProblemResults.projected\_gradient*.

Type *ndarray*

**hessian**

Estimated Hessian of the GMM objective. By default, this is computed with finite central differences after the optimization routine finishes.

Type *ndarray*

**reduced\_hessian**

Reduced Hessian of the GMM objective. When there are no parameter bounds, this will always be equal to *ProblemResults.hessian*. Otherwise, if an element in  $\hat{\theta}$  is equal to either its lower or upper bound, the corresponding row and column in the reduced Hessian will be all zeros.

Type *ndarray*

**reduced\_hessian\_eigenvalues**

Eigenvalues of *ProblemResults.reduced\_hessian*.

Type *ndarray*

**W**

Weighting matrix, *W*, used to compute these results.

Type *ndarray*

**updated\_W**

Weighting matrix updated according to (3.24).

Type *ndarray*

**Examples**

- *Tutorial*

**Methods**

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*bootstrap*([draws, seed, iteration])

Use a parametric bootstrap to create an empirical distribution of results.

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Continued on next page

Table 12 – continued from previous page

<code>compute_aggregate_elasticities([factor, ...])</code>	Estimate aggregate elasticities of demand, $\mathcal{E}$ , with respect to a variable, $x$ .
<code>compute_approximate_prices([firm_ids, ...])</code>	Approximate equilibrium prices after firm or cost changes, $p^*$ , under the assumption that shares and their price derivatives are unaffected by such changes.
<code>compute_consumer_surpluses([prices, ...])</code>	Estimate population-normalized consumer surpluses, CS.
<code>compute_costs([firm_ids, ownership, market_id])</code>	Estimate marginal costs, $c$ .
<code>compute_delta([agent_data, integration, ...])</code>	Estimate mean utilities, $\delta$ .
<code>compute_diversion_ratios([name, market_id])</code>	Estimate matrices of diversion ratios, $\mathcal{D}$ , with respect to a variable, $x$ .
<code>compute_elasticities([name, market_id])</code>	Estimate matrices of elasticities of demand, $\epsilon$ , with respect to a variable, $x$ .
<code>compute_hhi([firm_ids, shares, market_id])</code>	Estimate Herfindahl-Hirschman Indices, HHI.
<code>compute_long_run_diversion_ratios([market_id])</code>	Estimate matrices of long-run diversion ratios, $\mathcal{D}$ .
<code>compute_markups([prices, costs, market_id])</code>	Estimate markups, $\mathcal{M}$ .
<code>compute_optimal_instruments([method, draws, ...])</code>	Estimate feasible optimal or efficient instruments, $Z_D^{\text{opt}}$ and $Z_S^{\text{opt}}$ .
<code>compute_prices([firm_ids, ownership, costs, ...])</code>	Estimate equilibrium prices after firm or cost changes, $p^*$ .
<code>compute_probabilities([market_id])</code>	Estimate matrices of choice probabilities.
<code>compute_profits([prices, shares, costs, ...])</code>	Estimate population-normalized gross expected profits, $\pi$ .
<code>compute_shares([prices, delta, agent_data, ...])</code>	Estimate shares.
<code>extract_diagonal_means(matrices)</code>	Extract means of diagonals from stacked $J_t \times J_t$ matrices for each market $t$ .
<code>extract_diagonals(matrices)</code>	Extract diagonals from stacked $J_t \times J_t$ matrices for each market $t$ .
<code>importance_sampling(draws[, ar_constant, ...])</code>	Use importance sampling to construct nodes and weights for integration.
<code>run_distance_test(unrestricted)</code>	Test the validity of model restrictions with the distance test.
<code>run_hansen_test()</code>	Test the validity of overidentifying restrictions with the Hansen $J$ test.
<code>run_lm_test()</code>	Test the validity of model restrictions with the Lagrange multiplier test.
<code>run_wald_test(restrictions, ...)</code>	Test the validity of model restrictions with the Wald test.
<code>to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.

The results can be converted into a dictionary.

<code>ProblemResults.to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
---	--

### 5.5.2 pyblp.ProblemResults.to\_dict

`ProblemResults.to_dict` (*attributes*=(`'step'`, `'optimization_time'`, `'cumulative_optimization_time'`, `'total_time'`, `'cumulative_total_time'`, `'converged'`, `'cumulative_converged'`, `'optimization_iterations'`, `'cumulative_optimization_iterations'`, `'objective_evaluations'`, `'cumulative_objective_evaluations'`, `'fp_converged'`, `'cumulative_fp_converged'`, `'fp_iterations'`, `'cumulative_fp_iterations'`, `'contraction_evaluations'`, `'cumulative_contraction_evaluations'`, `'parameters'`, `'parameter_covariances'`, `'theta'`, `'sigma'`, `'sigma_squared'`, `'pi'`, `'rho'`, `'beta'`, `'gamma'`, `'sigma_se'`, `'sigma_squared_se'`, `'pi_se'`, `'rho_se'`, `'beta_se'`, `'gamma_se'`, `'sigma_bounds'`, `'pi_bounds'`, `'rho_bounds'`, `'beta_bounds'`, `'gamma_bounds'`, `'sigma_labels'`, `'pi_labels'`, `'rho_labels'`, `'beta_labels'`, `'gamma_labels'`, `'delta'`, `'tilde_costs'`, `'clipped_shares'`, `'clipped_costs'`, `'xi'`, `'omega'`, `'micro'`, `'objective'`, `'xi_by_theta_jacobian'`, `'omega_by_theta_jacobian'`, `'micro_by_theta_jacobian'`, `'gradient'`, `'projected_gradient'`, `'projected_gradient_norm'`, `'hessian'`, `'reduced_hessian'`, `'reduced_hessian_eigenvalues'`, `'W'`, `'updated_W'`))

Convert these results into a dictionary that maps attribute names to values.

Once converted to a dictionary, these results can be saved to a file with `pickle.dump()`.

**Parameters attributes** (*sequence of str, optional*) – Name of attributes that will be added to the dictionary. By default, all `ProblemResults` attributes are added except for `ProblemResults.problem` and `ProblemResults.last_results`.

**Returns** Mapping from attribute names to values.

**Return type** `dict`

#### Examples

- [Tutorial](#)

The following methods test the validity of overidentifying and model restrictions.

<code>ProblemResults.run_hansen_test()</code>	Test the validity of overidentifying restrictions with the Hansen $J$ test.
<code>ProblemResults.run_distance_test(unrestricted)</code>	Test the validity of model restrictions with the distance test.
<code>ProblemResults.run_lm_test()</code>	Test the validity of model restrictions with the Lagrange multiplier test.
<code>ProblemResults.run_wald_test(restrictions, ...)</code>	Test the validity of model restrictions with the Wald test.

### 5.5.3 pyblp.ProblemResults.run\_hansen\_test

`ProblemResults.run_hansen_test()`

Test the validity of overidentifying restrictions with the Hansen  $J$  test.

Following [Hansen \(1982\)](#), the  $J$  statistic is

$$J = N\bar{g}(\hat{\theta})'W\bar{g}(\hat{\theta}) \quad (5.13)$$

where  $\bar{g}(\hat{\theta})$  is defined in (3.11) and  $W$  is the optimal weighting matrix in (3.24).

---

**Note:** The statistic can equivalently be written as  $J = Nq(\hat{\theta})$  where the GMM objective value is defined in (3.10), or the same but without the  $N$  if the GMM objective value was scaled by  $N$ , which is the default behavior.

---

When the overidentifying restrictions in this model are valid, the  $J$  statistic is asymptotically  $\chi^2$  with degrees of freedom equal to the number of overidentifying restrictions. This requires that there are more moments than parameters.

**Warning:** This test requires `ProblemResults.W` to be an optimal weighting matrix, so it should typically be run only after two-step GMM or after one-step GMM with a pre-specified optimal weighting matrix.

**Returns** The  $J$  statistic.

**Return type** *float*

### Examples

- *Tutorial*

## 5.5.4 `pyblp.ProblemResults.run_distance_test`

`ProblemResults.run_distance_test` (*unrestricted*)

Test the validity of model restrictions with the distance test.

Following *Newey and West (1987)*, the distance or likelihood ratio-like statistic is

$$\text{LR} = J(\hat{\theta}^r) - J(\hat{\theta}^u) \quad (5.14)$$

where  $J(\hat{\theta}^r)$  is the  $J$  statistic defined in (5.13) for this restricted model and  $J(\hat{\theta}^u)$  is the  $J$  statistic for the unrestricted model.

---

**Note:** The statistic can equivalently be written as  $\text{LR} = N[q(\hat{\theta}^r) - q(\hat{\theta}^u)]$  where the GMM objective value is defined in (3.10), or the same but without the  $N$  if the GMM objective value was scaled by  $N$ , which is the default behavior.

---

If the restrictions in this model are valid, the distance statistic is asymptotically  $\chi^2$  with degrees of freedom equal to the number of restrictions.

**Warning:** This test requires each model's `ProblemResults.W` to be the optimal weighting matrix, so it should typically be run only after two-step GMM or after one-step GMM with pre-specified optimal weighting matrices.

**Parameters** `unrestricted` (`ProblemResults`) – `ProblemResults` for the unrestricted model.

**Returns** The distance statistic.

**Return type** *float*

## Examples

- [Tutorial](#)

### 5.5.5 `pyblp.ProblemResults.run_lm_test`

`ProblemResults.run_lm_test()`

Test the validity of model restrictions with the Lagrange multiplier test.

Following *Newey and West (1987)*, the Lagrange multiplier or score statistic is

$$\text{LM} = N\bar{g}(\hat{\theta})'W\bar{G}(\hat{\theta})V\bar{G}(\hat{\theta})'W\bar{g}(\hat{\theta}) \quad (5.15)$$

where  $\bar{g}(\hat{\theta})$  is defined in (3.11),  $\bar{G}(\hat{\theta})$  is defined in (3.19),  $W$  is the optimal weighting matrix in (3.24), and  $V$  is the covariance matrix of parameters in (3.30).

If the restrictions in this model are valid, the Lagrange multiplier statistic is asymptotically  $\chi^2$  with degrees of freedom equal to the number of restrictions.

**Warning:** This test requires `ProblemResults.W` to be an optimal weighting matrix, so it should typically be run only after two-step GMM or after one-step GMM with a pre-specified optimal weighting matrix.

**Returns** The Lagrange multiplier statistic.

**Return type** `float`

## Examples

- [Tutorial](#)

### 5.5.6 `pyblp.ProblemResults.run_wald_test`

`ProblemResults.run_wald_test(restrictions, restrictions_jacobian)`

Test the validity of model restrictions with the Wald test.

Following *Newey and West (1987)*, the Wald statistic is

$$\text{Wald} = Nr(\hat{\theta})'[R(\hat{\theta})VR(\hat{\theta})]^{-1}r(\hat{\theta}) \quad (5.16)$$

where the restrictions are  $r(\theta) = 0$  under the test's null hypothesis, their Jacobian is  $R(\theta) = \frac{\partial r(\theta)}{\partial \theta}$ , and  $V$  is the covariance matrix of parameters in (3.30).

If the restrictions are valid, the Wald statistic is asymptotically  $\chi^2$  with degrees of freedom equal to the number of restrictions.

#### Parameters

- **restrictions** (*array-like*) – Column vector of the model restrictions evaluated at the estimated parameters,  $r(\hat{\theta})$ .
- **restrictions\_jacobian** (*array-like*) – Estimated Jacobian of the restrictions with respect to all parameters,  $R(\hat{\theta})$ . This matrix should have as many rows as `restrictions` and as many columns as `ProblemResults.parameter_covariances`.

**Returns** The Wald statistic.

**Return type** `float`

## Examples

- *Tutorial*

In addition to class attributes, other post-estimation outputs can be estimated market-by-market with the following methods, which each return an array.

<code>ProblemResults.compute_aggregate_elasticities([firm_ids, ...])</code>	Estimate aggregate elasticities of demand, $\mathcal{E}$ , with respect to a variable, $x$ .
<code>ProblemResults.compute_elasticities([firm_ids, ...])</code>	Estimate matrices of elasticities of demand, $\varepsilon$ , with respect to a variable, $x$ .
<code>ProblemResults.compute_diversion_ratios([firm_ids, ...])</code>	Estimate matrices of diversion ratios, $\mathcal{D}$ , with respect to a variable, $x$ .
<code>ProblemResults.compute_long_run_diversion_ratios([firm_ids, ...])</code>	Estimate matrices of long-run diversion ratios, $\mathcal{D}$ .
<code>ProblemResults.compute_probabilities([firm_ids, ...])</code>	Estimate matrices of choice probabilities.
<code>ProblemResults.extract_diagonals(matrices)</code>	Extract diagonals from stacked $J_t \times J_t$ matrices for each market $t$ .
<code>ProblemResults.extract_diagonal_means(matrices)</code>	Estimate means of diagonals from stacked $J_t \times J_t$ matrices for each market $t$ .
<code>ProblemResults.compute_delta([agent_data, ...])</code>	Estimate mean utilities, $\delta$ .
<code>ProblemResults.compute_costs([firm_ids, ...])</code>	Estimate marginal costs, $c$ .
<code>ProblemResults.compute_approximate_prices([firm_ids, ...])</code>	Approximate equilibrium prices after firm or cost changes, $p^*$ , under the assumption that shares and their price derivatives are unaffected by such changes.
<code>ProblemResults.compute_prices([firm_ids, ...])</code>	Estimate equilibrium prices after firm or cost changes, $p^*$ .
<code>ProblemResults.compute_shares([prices, ...])</code>	Estimate shares.
<code>ProblemResults.compute_hhi([firm_ids, ...])</code>	Estimate Herfindahl-Hirschman Indices, HHI.
<code>ProblemResults.compute_markup([prices, ...])</code>	Estimate markups, $\mathcal{M}$ .
<code>ProblemResults.compute_profits([prices, ...])</code>	Estimate population-normalized gross expected profits, $\pi$ .
<code>ProblemResults.compute_consumer_surplus([prices, ...])</code>	Estimate population-normalized consumer surpluses, CS.

### 5.5.7 pyblp.ProblemResults.compute\_aggregate\_elasticities

`ProblemResults.compute_aggregate_elasticities` (*factor=0.1*, *name='prices'*, *market\_id=None*)

Estimate aggregate elasticities of demand,  $\mathcal{E}$ , with respect to a variable,  $x$ .

In market  $t$ , the aggregate elasticity of demand is

$$\mathcal{E} = \sum_{j \in J_t} \frac{s_{jt}(x + \Delta x) - s_{jt}}{\Delta}, \quad (5.17)$$

in which  $\Delta$  is a scalar factor and  $s_{jt}(x + \Delta x)$  is the share of product  $j$  in market  $t$ , evaluated at the scaled values of the variable.

#### Parameters

- **factor** (*float, optional*) – The scalar factor,  $\Delta$ .
- **name** (*str, optional*) – Name of the variable,  $x$ . By default,  $x = p$ , prices.
- **market\_id** (*object, optional*) – ID of the market in which to compute aggregate elasticities. By default, aggregate elasticities are computed in all markets and stacked.

**Returns** Estimates of aggregate elasticities of demand,  $\mathcal{E}$ . If `market_id` was not specified, rows are in the same order as `Problem.unique_market_ids`.

**Return type** `ndarray`

### Examples

- [Tutorial](#)

## 5.5.8 `pyblp.ProblemResults.compute_elasticities`

`ProblemResults.compute_elasticities` (*name='prices', market\_id=None*)

Estimate matrices of elasticities of demand,  $\varepsilon$ , with respect to a variable,  $x$ .

In market  $t$ , the value in row  $j$  and column  $k$  of  $\varepsilon$  is

$$\varepsilon_{jk} = \frac{x_{kt}}{s_{jt}} \frac{\partial s_{jt}}{\partial x_{kt}}. \quad (5.18)$$

### Parameters

- **name** (*str, optional*) – Name of the variable,  $x$ . By default,  $x = p$ , prices.
- **market\_id** (*object, optional*) – ID of the market in which to compute elasticities. By default, elasticities are computed in all markets and stacked.

**Returns** Estimated  $J_t \times J_t$  matrices of elasticities of demand,  $\varepsilon$ . If `market_id` was not specified, matrices are estimated in each market  $t$  and stacked. Columns for a market are in the same order as products for the market. If a market has fewer products than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

### Examples

- [Tutorial](#)

## 5.5.9 `pyblp.ProblemResults.compute_diversion_ratios`

`ProblemResults.compute_diversion_ratios` (*name='prices', market\_id=None*)

Estimate matrices of diversion ratios,  $\mathcal{D}$ , with respect to a variable,  $x$ .

In market  $t$ , the value in row  $j$  and column  $k \neq j$  is

$$\mathcal{D}_{jk} = - \frac{\partial s_{kt}}{\partial x_{jt}} \bigg/ \frac{\partial s_{jt}}{\partial x_{jt}}. \quad (5.19)$$

Diversion ratios for the outside good are reported on diagonals:

$$\mathcal{D}_{jj} = - \frac{\partial s_{0t}}{\partial x_{jt}} \bigg/ \frac{\partial s_{jt}}{\partial x_{jt}}. \quad (5.20)$$



Unlike `ProblemResults.compute_long_run_diversion_ratios()`, this gives the marginal treatment effect (MTE) version of the diversion ratio. For more information, see [Conlon and Mortimer \(2018\)](#).

#### Parameters

- **name** (*str, optional*) – Name of the variable,  $x$ . By default,  $x = p$ , prices.
- **market\_id** (*object, optional*) – ID of the market in which to compute diversion ratios. By default, diversion ratios are computed in all markets and stacked.

**Returns** Estimated  $J_t \times J_t$  matrices of diversion ratios,  $\mathcal{D}$ . If `market_id` was not specified, matrices are estimated in each market  $t$  and stacked. Columns for a market are in the same order as products for the market. If a market has fewer products than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

#### Examples

- [Tutorial](#)

### 5.5.10 `pyblp.ProblemResults.compute_long_run_diversion_ratios`

`ProblemResults.compute_long_run_diversion_ratios` (*market\_id=None*)

Estimate matrices of long-run diversion ratios,  $\mathcal{D}$ .

In market  $t$ , the value in row  $j$  and column  $k \neq j$  is

$$\bar{\mathcal{D}}_{jk} = \frac{s_{k(-j)t} - s_{kt}}{s_{jt}}, \quad (5.21)$$

in which  $s_{k(-j)t}$  is the share of product  $k$  computed with  $j$  removed from the choice set. Long-run diversion ratios for the outside good are reported on diagonals:

$$\bar{\mathcal{D}}_{jj} = \frac{s_{0(-j)t} - s_0}{s_{jt}}. \quad (5.22)$$

Unlike `ProblemResults.compute_diversion_ratios()`, this gives the average treatment effect (ATE) version of the diversion ratio. For more information, see [Conlon and Mortimer \(2018\)](#).

**Parameters** **market\_id** (*object, optional*) – ID of the market in which to compute long-run diversion ratios. By default, long-run diversion ratios are computed in all markets and stacked.

**Returns** Estimated  $J_t \times J_t$  matrices of long-run diversion ratios,  $\mathcal{D}$ . If `market_id` was not specified, matrices are estimated in each market  $t$  and stacked. Columns for a market are in the same order as products for the market. If a market has fewer products than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

#### Examples

- [Tutorial](#)

### 5.5.11 `pyblp.ProblemResults.compute_probabilities`

`ProblemResults.compute_probabilities` (*market\_id=None*)

Estimate matrices of choice probabilities.

For each market, the value in row  $j$  and column  $i$  is given by (3.5) when there are random coefficients, and by (3.37) when there is additionally a nested structure. For the logit and nested logit models, choice probabilities are market shares.

**Parameters** `market_id` (*object, optional*) – ID of the market in which to compute choice probabilities. By default, choice probabilities are computed in all markets and stacked.

**Returns** Estimated  $J_t \times I_t$  matrices of choice probabilities. If `market_id` was not specified, matrices are estimated in each market  $t$  and stacked. Columns for a market are in the same order as agents for the market. If a market has fewer agents than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

#### Examples

- [Tutorial](#)

### 5.5.12 `pyblp.ProblemResults.extract_diagonals`

`ProblemResults.extract_diagonals` (*matrices*)

Extract diagonals from stacked  $J_t \times J_t$  matrices for each market  $t$ .

**Parameters** `matrices` (*array-like*) – Stacked matrices, such as estimates of  $\varepsilon$ , computed by `ProblemResults.compute_elasticities()`;  $\mathcal{D}$ , computed by `ProblemResults.compute_diversion_ratios()`;  $\bar{\mathcal{D}}$ , computed by `ProblemResults.compute_long_run_diversion_ratios()`; or  $s_{ijt}$  computed by `ProblemResults.compute_probabilities()`.

**Returns** Stacked matrix diagonals. If the matrices are estimates of  $\varepsilon$ , a diagonal is a market's own elasticities of demand; if they are estimates of  $\mathcal{D}$  or  $\bar{\mathcal{D}}$ , a diagonal is a market's diversion ratios to the outside good.

**Return type** `ndarray`

#### Examples

- [Tutorial](#)

### 5.5.13 `pyblp.ProblemResults.extract_diagonal_means`

`ProblemResults.extract_diagonal_means` (*matrices*)

Extract means of diagonals from stacked  $J_t \times J_t$  matrices for each market  $t$ .

**Parameters** `matrices` (*array-like*) – Stacked matrices, such as estimates of  $\varepsilon$ , computed by `ProblemResults.compute_elasticities()`;  $\mathcal{D}$ , computed by `ProblemResults.compute_diversion_ratios()`;  $\bar{\mathcal{D}}$ , computed by `ProblemResults.compute_long_run_diversion_ratios()`; or  $s_{ijt}$  computed by `ProblemResults.compute_probabilities()`.

**Returns** Stacked diagonal means. If the matrices are estimates of  $\varepsilon$ , the mean of a diagonal is a market's mean own elasticity of demand; if they are estimates of  $\mathcal{D}$  or  $\bar{\mathcal{D}}$ , the mean of a diagonal is a market's mean diversion ratio to the outside good. Rows are in the same order as `Problem.unique_market_ids`.

**Return type** `ndarray`

## Examples

- *Tutorial*

### 5.5.14 `pyblp.ProblemResults.compute_delta`

`ProblemResults.compute_delta` (*agent\_data=None, integration=None, iteration=None, fp\_type=None, shares\_bounds=(1e-300, None), market\_id=None*)

Estimate mean utilities,  $\delta$ .

This method can be used to compute mean utilities at the estimated parameters with a different integration configuration or with different fixed point iteration settings than those used during estimation. The estimated `ProblemResults.delta` will be used as starting values for the fixed point routine.

A more precisely estimated mean utility can be used, for example, by `ProblemResults.importance_sampling()`. It can also be used to `ProblemResults.compute_shares()` to compare the performance of different integration routines.

#### Parameters

- **agent\_data** (*structured array-like, optional*) – Agent data that will be used to compute  $\delta$ . By default, `agent_data` in `Problem` is used. For more information, refer to `Problem`.
- **integration** (*Integration, optional*) – `Integration` configuration that will be used to compute  $\delta$ , which will replace any `nodes` field in `agent_data`. This configuration is required if `agent_data` is specified without a `nodes` field. By default, `agent_data` in `Problem` is used. For more information, refer to `Problem`.
- **iteration** (*Iteration, optional*) – `Iteration` configuration for how to solve the fixed point problem used to compute  $\delta$  in each market. By default, `iteration` in `Problem.solve()` is used. For more information, refer to `Problem.solve()`.
- **fp\_type** (*str, optional*) – Configuration for the type of contraction mapping used to compute  $\delta$  in each market. By default, `fp_type` in `Problem.solve()` is used. For more information, refer to `Problem.solve()`.
- **shares\_bounds** (*tuple, optional*) – Configuration for  $s_{jt}(\delta, \theta)$  bounds of the form `(lb, ub)`, in which both `lb` and `ub` are floats or `None`. By default, simulated shares are bounded from below by `1e-300`. This is only relevant if `fp_type` is `'safe_linear'` or `'linear'`. Bounding shares in the contraction does nothing with a nonlinear fixed point. For more information, refer to `Problem.solve()`.
- **market\_id** (*object, optional*) – ID of the market in which to compute mean utilities. By default, mean utilities is computed in all markets and stacked.

**Returns** Mean utilities,  $\delta$ .

**Return type** `ndarray`

## Examples

- [Tutorial](#)

### 5.5.15 `pyblp.ProblemResults.compute_costs`

`ProblemResults.compute_costs` (*firm\_ids=None, ownership=None, market\_id=None*)

Estimate marginal costs,  $c$ .

Marginal costs are computed with the  $\eta$ -markup equation in (3.7):

$$c = p - \eta. \quad (5.23)$$

#### Parameters

- **firm\_ids** (*array-like, optional*) – Firm IDs. By default, the `firm_ids` field of `product_data` in `Problem` will be used.
- **ownership** (*array-like, optional*) – Ownership matrices. By default, standard ownership matrices based on `firm_ids` will be used unless the `ownership` field of `product_data` in `Problem` was specified.
- **market\_id** (*object, optional*) – ID of the market in which to compute marginal costs. By default, marginal costs are computed in all markets and stacked.

**Returns** Marginal costs,  $c$ .

**Return type** `ndarray`

## Examples

- [Tutorial](#)

### 5.5.16 `pyblp.ProblemResults.compute_approximate_prices`

`ProblemResults.compute_approximate_prices` (*firm\_ids=None, ownership=None, costs=None, market\_id=None*)

Approximate equilibrium prices after firm or cost changes,  $p^*$ , under the assumption that shares and their price derivatives are unaffected by such changes.

This approximation is discussed in, for example, *Nevo (1997)*. Prices in each market are computed according to the  $\eta$ -markup equation in (3.7):

$$p^* = c^* + \eta^*, \quad (5.24)$$

in which the markup term is approximated with

$$\eta^* \approx - \left( \mathcal{H}^* \odot \frac{\partial s}{\partial p} \right)^{-1} s \quad (5.25)$$

where  $\mathcal{H}^*$  is the ownership or product holding matrix associated with firm changes.

#### Parameters

- **firm\_ids** (*array-like, optional*) – Potentially changed firm IDs. By default, the unchanged `firm_ids` field of `product_data` in `Problem` will be used.

- **ownership** (*array-like, optional*) – Potentially changed ownership matrices. By default, standard ownership matrices based on `firm_ids` will be used unless the `ownership` field of `product_data` in `Problem` was specified.
- **costs** (*array-like, optional*) – Potentially changed marginal costs,  $c^*$ . By default, unchanged marginal costs are computed with `ProblemResults.compute_costs()`. Costs under a changed ownership structure can be computed by specifying the `firm_ids` or `ownership` arguments of `ProblemResults.compute_costs()`.
- **market\_id** (*object, optional*) – ID of the market in which to compute approximate equilibrium prices. By default, approximate equilibrium prices are computed in all markets and stacked.

**Returns** Approximation of equilibrium prices after any firm or cost changes,  $p^*$ .

**Return type** `ndarray`

### Examples

- [Tutorial](#)

## 5.5.17 `pyblp.ProblemResults.compute_prices`

`ProblemResults.compute_prices` (*firm\_ids=None, ownership=None, costs=None, prices=None, iteration=None, market\_id=None*)

Estimate equilibrium prices after firm or cost changes,  $p^*$ .

---

**Note:** To compute equilibrium prices (and shares) associated with a more complicated counterfactual, a `Simulation` for the counterfactual can be initialized with the estimated parameters, structural errors, and marginal costs from these results, and then solved with `Simulation.replace_endogenous()`. The returned `SimulationResults` gives more information about the contraction than this method, such as the number of contraction evaluations.

---

Prices are computed in each market by iterating over the  $\zeta$ -markup contraction in (3.46):

$$p^* \leftarrow c^* + \zeta^*(p^*), \quad (5.26)$$

in which the markup term from (3.43) is

$$\zeta^*(p^*) = \Lambda^{-1}(p^*)[\mathcal{H}^* \odot \Gamma(p^*)]'(p^* - c^*) - \Lambda^{-1}(p^*)s(p^*) \quad (5.27)$$

where  $\mathcal{H}^*$  is the ownership matrix associated with firm changes.

**Warning:** This routine assumes that marginal costs,  $c$ , remain constant. This may not be the case if `shares` was included in the formulation for  $X_3$  in `Problem`.

### Parameters

- **firm\_ids** (*array-like, optional*) – Potentially changed firm IDs. By default, the unchanged `firm_ids` field of `product_data` in `Problem` will be used.
- **ownership** (*array-like, optional*) – Potentially changed ownership matrices. By default, standard ownership matrices based on `firm_ids` will be used unless the `ownership` field of `product_data` in `Problem` was specified.

- **costs** (*array-like*) – Potentially changed marginal costs,  $c^*$ . By default, unchanged marginal costs are computed with `ProblemResults.compute_costs()`. Costs under a changed ownership structure can be computed by specifying the `firm_ids` or `ownership` arguments of `ProblemResults.compute_costs()`. If marginal costs depend on prices through market shares, they will be updated to reflect different prices during each iteration of the routine. Updated marginal costs can be obtained by instead using `Simulation.replace_endogenous()`.
- **prices** (*array-like, optional*) – Prices at which the fixed point iteration routine will start. By default, unchanged prices,  $p$ , are used as starting values. Other reasonable starting prices include the approximate equilibrium prices computed by `ProblemResults.compute_approximate_prices()`.
- **iteration** (*Iteration, optional*) – `Iteration` configuration for how to solve the fixed point problem in each market. By default, `Iteration('simple', {'atol': 1e-12})` is used. Analytic Jacobians are not supported for solving this system.
- **market\_id** (*object, optional*) – ID of the market in which to compute equilibrium prices. By default, equilibrium prices are computed in all markets and stacked.

**Returns** Estimates of equilibrium prices after any firm or cost changes,  $p^*$ .

**Return type** `ndarray`

## Examples

- [Tutorial](#)

### 5.5.18 `pyblp.ProblemResults.compute_shares`

`ProblemResults.compute_shares` (*prices=None, delta=None, agent\_data=None, integration=None, market\_id=None*)

Estimate shares.

It may be desirable to compute the shares associated with equilibrium prices that have been computed, for example, by `ProblemResults.compute_prices()`.

---

**Note:** To compute equilibrium shares (and prices) associated with a more complicated counterfactual, a `Simulation` for the counterfactual can be initialized with the estimated parameters, structural errors, and marginal costs from these results, and then solved with `Simulation.replace_endogenous()`.

---

Alternatively, this method can also be used to evaluate the performance of different numerical integration configurations. One way to do so is to use `ProblemResults.compute_delta()` to compute mean utilities with a very precise integration rule (one that is infeasible to use during estimation), use these same mean utilities and integration rule to precisely compute shares, and then compare error between these precisely-computed shares and shares computed with less precise (but feasible to use during estimation) integration rules, still using the precisely-computed mean utilities.

#### Parameters

- **prices** (*array-like, optional*) – Prices at which to evaluate shares, such as equilibrium prices,  $p^*$ , computed by `ProblemResults.compute_prices()`. By default, unchanged prices are used.

- **delta** (*array-like, optional*) – Mean utilities that will be used to evaluate shares, such as those computed more precisely by `ProblemResults.compute_delta()`. By default, the estimated `ProblemResults.delta` is used, and updated with any specified prices.
- **agent\_data** (*structured array-like, optional*) – Agent data that will be used to compute shares. By default, `agent_data` in `Problem` is used. For more information, refer to `Problem`.
- **integration** (*Integration, optional*) – `Integration` configuration that will be used to compute shares, which will replace any `nodes` field in `agent_data`. This configuration is required if `agent_data` is specified without a `nodes` field. By default, `agent_data` in `Problem` is used. For more information, refer to `Problem`.
- **market\_id** (*object, optional*) – ID of the market in which to compute shares. By default, shares are computed in all markets and stacked.

**Returns** Estimates of shares evaluated at the specified prices.

**Return type** `ndarray`

### Examples

- *Tutorial*

#### 5.5.19 `pyblp.ProblemResults.compute_hhi`

`ProblemResults.compute_hhi` (*firm\_ids=None, shares=None, market\_id=None*)

Estimate Herfindahl-Hirschman Indices, HHI.

The index in market  $t$  is

$$\text{HHI} = 10,000 \times \sum_{f \in F_t} \left( \sum_{j \in J_{ft}} s_{jt} \right)^2. \quad (5.28)$$

#### Parameters

- **firm\_ids** (*array-like, optional*) – Firm IDs. By default, the unchanged `firm_ids` field of `product_data` in `Problem` will be used.
- **shares** (*array-like, optional*) – Shares,  $s$ , such as those computed by `ProblemResults.compute_shares()`. By default, unchanged shares are used.
- **market\_id** (*object, optional*) – ID of the market in which to compute the index. By default, indices are computed in all markets and stacked.

**Returns** Estimated Herfindahl-Hirschman Indices, HHI. If `market_ids` was not specified, rows are in the same order as `Problem.unique_market_ids`.

**Return type** `ndarray`

### Examples

- *Tutorial*

### 5.5.20 `pyblp.ProblemResults.compute_markup`s

`ProblemResults.compute_markup`s (*prices=None, costs=None, market\_id=None*)

Estimate markups,  $\mathcal{M}$ .

The markup of product  $j$  in market  $t$  is

$$\mathcal{M}_{jt} = \frac{p_{jt} - c_{jt}}{p_{jt}}. \quad (5.29)$$

#### Parameters

- **prices** (*array-like, optional*) – Prices,  $p$ , such as equilibrium prices,  $p^*$ , computed by `ProblemResults.compute_prices()`. By default, unchanged prices are used.
- **costs** (*array-like*) – Marginal costs,  $c$ . By default, marginal costs are computed with `ProblemResults.compute_costs()`. Costs under a changed ownership structure can be computed by specifying the `firm_ids` or `ownership` arguments of `ProblemResults.compute_costs()`.
- **market\_id** (*object, optional*) – ID of the market in which to compute markups. By default, markups are computed in all markets and stacked.

**Returns** Estimated markups,  $\mathcal{M}$ .

**Return type** `ndarray`

#### Examples

- [Tutorial](#)

### 5.5.21 `pyblp.ProblemResults.compute_profits`

`ProblemResults.compute_profits` (*prices=None, shares=None, costs=None, market\_id=None*)

Estimate population-normalized gross expected profits,  $\pi$ .

With constant costs, the profit from product  $j$  in market  $t$  is

$$\pi_{jt} = (p_{jt} - c_{jt})s_{jt}. \quad (5.30)$$

#### Parameters

- **prices** (*array-like, optional*) – Prices,  $p$ , such as equilibrium prices,  $p^*$ , computed by `ProblemResults.compute_prices()`. By default, unchanged prices are used.
- **shares** (*array-like, optional*) – Shares,  $s$ , such as those computed by `ProblemResults.compute_shares()`. By default, unchanged shares are used.
- **costs** (*array-like*) – Marginal costs,  $c$ . By default, marginal costs are computed with `ProblemResults.compute_costs()`. Costs under a changed ownership structure can be computed by specifying the `firm_ids` or `ownership` arguments of `ProblemResults.compute_costs()`.
- **market\_id** (*object, optional*) – ID of the market in which to compute profits. By default, profits are computed in all markets and stacked.

**Returns** Estimated population-normalized gross expected profits,  $\pi$ .

**Return type** `ndarray`



## Examples

- *Tutorial*

### 5.5.22 `pyblp.ProblemResults.compute_consumer_surpluses`

`ProblemResults.compute_consumer_surpluses` (*prices=None*, *market\_id=None*,  
*keep\_all=False*)

Estimate population-normalized consumer surpluses, CS.

Assuming away nonlinear income effects, the surplus in market  $t$  is

$$CS = \sum_{i \in I_t} w_{it} CS_{it}, \quad (5.31)$$

in which the consumer surplus for individual  $i$  is

$$CS_{it} = \log \left( 1 + \sum_{j \in J_t} \exp V_{ijt} \right) / \left( -\frac{\partial V_{i1t}}{\partial p_{1t}} \right), \quad (5.32)$$

or with nesting parameters,

$$CS_{it} = \log \left( 1 + \sum_{h \in H} \exp V_{iht} \right) / \left( -\frac{\partial V_{i1t}}{\partial p_{1t}} \right) \quad (5.33)$$

where  $V_{ijt}$  is defined in (3.1) and  $V_{iht}$  is defined in (3.38).

**Warning:**  $\frac{\partial V_{i1t}}{\partial p_{1t}}$  is the derivative of utility for the first product with respect to its price. The first product is chosen arbitrarily because this method assumes that there are no nonlinear income effects, which implies that this derivative is the same for all products. Computed consumer surpluses will likely be incorrect if prices are formulated in a nonlinear fashion like `log(prices)`.

#### Parameters

- **prices** (*array-like, optional*) – Prices at which utilities and price derivatives will be evaluated, such as equilibrium prices,  $p^*$ , computed by `ProblemResults.compute_prices()`. By default, unchanged prices are used.
- **market\_id** (*object, optional*) – ID of the market in which to compute consumer surplus. By default, consumer surpluses are computed in all markets and stacked.
- **keep\_all** (*bool, optional*) – Whether to keep all individuals' surpluses  $CS_{it}$  or just market-level surpluses. By default only market-level surpluses are returned, but returning all surpluses will be important for analysis by agent type or demographic category.

**Returns** Estimated population-normalized consumer surpluses, CS (or individuals' surpluses if `keep_all` is `True`). If `market_ids` was not specified, rows are in the same order as `Problem.unique_market_ids`. If `keep_all` is `True`, columns for a market are in the same order as agents for the market. If a market has fewer agents than others, extra columns will contain `numpy.nan`.

**Return type** `ndarray`

## Examples

- *Tutorial*

A parametric bootstrap can be used, for example, to compute standard errors for the above post-estimation outputs. The following method returns a results class with all of the above methods, which returns a distribution of post-estimation outputs corresponding to different bootstrapped samples.

---

<code><i>ProblemResults.bootstrap</i>([draws, seed, ...])</code>	Use a parametric bootstrap to create an empirical distribution of results.
--	--

---

### 5.5.23 `pyblp.ProblemResults.bootstrap`

`ProblemResults.bootstrap` (*draws=1000, seed=None, iteration=None*)

Use a parametric bootstrap to create an empirical distribution of results.

The constructed `BootstrappedResults` can be used just like `ProblemResults` to compute various post-estimation outputs for different markets. The only difference is that `BootstrappedResults` methods return arrays with an extra first dimension, along which bootstrapped results are stacked. These stacked results can be used to construct, for example, confidence intervals for post-estimation outputs.

For each bootstrap draw, parameters are drawn from the estimated multivariate normal distribution of all parameters defined by `ProblemResults.parameters` and `ProblemResults.parameter_covariances`. Any bounds configured in `Problem.solve()` will also bound parameter draws. Each parameter draw is used to compute the implied mean utility,  $\delta$ , and shares,  $s$ . If a supply side was estimated, the implied marginal costs,  $c$ , and prices,  $p$ , are computed as well by iterating over the  $\zeta$ -markup contraction in (3.46). If marginal costs depend on prices through market shares, they will be updated to reflect different prices during each iteration of the routine.

---

**Note:** By default, parametric bootstrapping may use a lot of memory. This is because all bootstrapped results (for all `draws`) are stored in memory at the same time. Memory usage can be reduced by calling this method in a loop with `draws = 1`. In each iteration of the loop, compute the desired post-estimation output with the proper method of the returned `BootstrappedResults` class and store these outputs.

---

#### Parameters

- **draws** (*int, optional*) – The number of draws that will be taken from the joint distribution of the parameters. The default value is 1000.
- **seed** (*int, optional*) – Passed to `numpy.random.RandomState` to seed the random number generator before any draws are taken. By default, a seed is not passed to the random number generator.
- **iteration** (*Iteration, optional*) – `Iteration` configuration used to compute bootstrapped prices by iterating over the  $\zeta$ -markup equation in (3.46). By default, if a supply side was estimated, this is `Iteration('simple', {'atol': 1e-12})`. Analytic Jacobians are not supported for solving this system. This configuration is not used if a supply side was not estimated.

**Returns** Computed `BootstrappedResults`.

**Return type** `BootstrappedResults`

## Examples

- *Tutorial*

Optimal instruments, which also return a results class instead of an array, can be estimated with the following method.

---

`ProblemResults.compute_optimal_instruments` (Estimate feasible optimal or efficient instruments,  $Z_D^{\text{opt}}$  and  $Z_S^{\text{opt}}$ .)

---

### 5.5.24 `pyblp.ProblemResults.compute_optimal_instruments`

`ProblemResults.compute_optimal_instruments` (*method='approximate', draws=1, seed=None, expected\_prices=None, iteration=None*)

Estimate feasible optimal or efficient instruments,  $Z_D^{\text{opt}}$  and  $Z_S^{\text{opt}}$ .

Optimal instruments have been shown, for example, by *Reynaert and Verboven (2014)* and *Conlon and Gortmaker (2020)*, to reduce bias, improve efficiency, and enhance stability of BLP estimates.

Optimal instruments in the spirit of *Amemiya (1977)* or *Chamberlain (1987)* are defined by

$$\begin{bmatrix} Z_{D,jt}^{\text{opt}} \\ Z_{S,jt}^{\text{opt}} \end{bmatrix} = \Sigma_{\xi\omega}^{-1} E \left[ \begin{array}{c} \frac{\partial \xi_{jt}}{\partial \theta} \\ \frac{\partial \omega_{jt}}{\partial \theta} \end{array} \middle| Z \right], \quad (5.34)$$

in which  $Z$  are all exogenous variables.

Feasible optimal instruments are estimated by evaluating this expression at an estimated  $\hat{\theta}$ . The expectation is taken by approximating an integral over the joint density of  $\xi$  and  $\omega$ . For each error term realization, if not already estimated, equilibrium prices and shares are computed by iterating over the  $\zeta$ -markup contraction in (3.46). If marginal costs depend on prices through market shares, they will be updated to reflect different prices during each iteration of the routine.

The expected Jacobians are estimated with the average over all computed Jacobian realizations. The  $2 \times 2$  normalizing matrix  $\Sigma_{\xi\omega}$  is estimated with the sample covariance matrix of the error terms.

Optimal instruments for linear parameters not included in  $\theta$  are simple product characteristics, so they are not computed here but are rather included in the final set of instruments by `OptimalInstrumentResults.to_problem()`.

---

**Note:** When both a supply and demand side are estimated, there are usually collinear rows in (5.34) because of overlapping product characteristics in  $X_1$  and  $X_3$ . The expression can be corrected by multiplying it with a conformable matrix of ones and zeros that remove the collinearity problem. The question of which rows to exclude is addressed in `OptimalInstrumentResults.to_problem()`.

---

#### Parameters

- **method** (*str, optional*) – The method by which the integral over the joint density of  $\xi$  and  $\omega$  is approximated. The following methods are supported:
  - 'approximate' (default) - Evaluate the Jacobians at the expected value of the error terms: zero (draws will be ignored).
  - 'normal' - Draw from the normal approximation to the joint distribution of the error terms and take the average over the computed Jacobians (draws determines the number of draws).

- 'empirical' - Draw with replacement from the empirical joint distribution of the error terms and take the average over the computed Jacobians (draws determines the number of draws).
- **draws** (*int, optional*) – The number of draws that will be taken from the joint distribution of the error terms. This is ignored if method is 'approximate'. Because the default method is 'approximate', the default number of draws is 1, even though it will be ignored. For 'normal' or empirical, larger numbers such as 100 or 1000 are recommended.
- **seed** (*int, optional*) – Passed to `numpy.random.RandomState` to seed the random number generator before any draws are taken. By default, a seed is not passed to the random number generator.
- **expected\_prices** (*array-like, optional*) – Vector of expected prices conditional on all exogenous variables,  $E[p | Z]$ . By default, if a supply side was estimated and `shares` did not enter into the formulation for  $X_3$  in *Problem*, `iteration` is used. Otherwise, this is by default estimated with the fitted values from a reduced form regression of endogenous prices onto  $Z_D$ .
- **iteration** (*Iteration, optional*) – *Iteration* configuration used to estimate expected prices by iterating over the  $\zeta$ -markup contraction in (3.46). By default, if a supply side was estimated, this is `Iteration('simple', {'atol': 1e-12})`. Analytic Jacobians are not supported for solving this system. This configuration is not used if `expected_prices` is specified.

**Returns** Computed *OptimalInstrumentResults*.

**Return type** *OptimalInstrumentResults*

## Examples

- *Tutorial*

Importance sampling can be used to create new integration nodes and weights. Its method also returns a results class.

---

```
ProblemResults.importance_sampling(draws[Use importance sampling to construct nodes and
...])
```

---

weights for integration.

### 5.5.25 `pyblp.ProblemResults.importance_sampling`

`ProblemResults.importance_sampling` (*draws, ar\_constant=1.0, seed=None, agent\_data=None, integration=None, delta=None*)

Use importance sampling to construct nodes and weights for integration.

Importance sampling is done with the accept/reject procedure of *Berry, Levinsohn, and Pakes (1995)*. First, `agent_data` and/or `integration` are used to provide a large number of candidate sampling nodes  $\nu$  and any demographics  $d$ .

Out of these candidate agent data, each candidate agent  $i$  in market  $t$  is accepted with probability  $\frac{1-s_{i0t}}{M}$  where  $M \geq 1$  is some accept-reject constant. The probability of choosing an inside good  $1 - s_{i0t}$ , is evaluated at the estimated  $\hat{\theta}$  and  $\delta(\hat{\theta})$ .

Optionally, `ProblemResults.compute_delta()` can be used to provide a more precise  $\delta(\hat{\theta})$  than the estimated `ProblemResults.delta`. The idea is that more precise agent data (i.e., more integration nodes) would be infeasible to use during estimation, but is feasible here because  $\delta(\hat{\theta})$  only needs to be computed once

given a  $\hat{\theta}$ .

Out of the remaining accepted agents,  $I_t$  equal to draws are randomly selected within each market  $t$  and assigned integration weights  $w_{it} = \frac{1}{I_t} \cdot \frac{1-s_{0t}}{1-s_{i0t}}$ .

If this procedure accepts fewer than draws agents in a market, an exception will be raised. A good rule of thumb is to provide more candidate draws in each market than  $\frac{M \times I_t}{1-s_{0t}}$ .

### Parameters

- **draws** (*int, optional*) – Number of draws to take from `sampling_agent_data` in each market.
- **ar\_constant** (*float, optional*) – Accept/reject constant  $M \geq 1$ , which is by default, 1.0.
- **seed** (*int, optional*) – Passed to `numpy.random.RandomState` to seed the random number generator before importance sampling is done. By default, a seed is not passed to the random number generator.
- **agent\_data** (*structured array-like, optional*) – Agent data from which draws will be sampled, which should have the same structure as `agent_data` in `Problem`. The `weights` field does not need to be specified, and if it is specified it will be ignored. By default, the same agent data used to solve the problem will be used.
- **integration** (*Integration, optional*) – `Integration` configuration for how to build nodes from which draws will be sampled, which will replace any `nodes` field in `sampling_agent_data`. This configuration is required if `sampling_agent_data` is specified without a `nodes` field.
- **delta** (*array-like, optional*) – More precise  $\delta(\hat{\theta})$  than the estimated `ProblemResults.delta`, which can be computed by passing a more precise integration rule to `ProblemResults.compute_delta()`. By default, `ProblemResults.delta` is used.

**Returns** Computed `ImportanceSamplingResults`.

**Return type** `ImportanceSamplingResults`

### Examples

- [Tutorial](#)

## 5.6 Bootstrapped Problem Results Class

Parametric bootstrap computation returns the following class.

---

`BootstrappedResults`

Bootstrapped results of a solved problem.

---

### 5.6.1 pyblp.BootstrappedResults

**class** `pyblp.BootstrappedResults`

Bootstrapped results of a solved problem.

This class has same methods as `ProblemResults` that compute post-estimation outputs in one or more markets, but not other methods like `ProblemResults.compute_optimal_instruments()` that do not make sense in a bootstrapped dataset. The only other difference is that methods return arrays with an

extra first dimension along which bootstrapped results are stacked (these stacked results can be used to construct, for example, confidence intervals for post-estimation outputs). Similarly, arrays of data (except for firm IDs and ownership matrices) passed as arguments to methods should have an extra first dimension of size `BootstrappedResults.draws`.

**problem\_results**

`ProblemResults` that was used to compute these bootstrapped results.

Type `ProblemResults`

**bootstrapped\_sigma**

Bootstrapped Cholesky decomposition of the covariance matrix for unobserved taste heterogeneity,  $\Sigma$ .

Type `ndarray`

**bootstrapped\_pi**

Bootstrapped parameters that measures how agent tastes vary with demographics,  $\Pi$ .

Type `ndarray`

**bootstrapped\_rho**

Bootstrapped parameters that measure within nesting group correlations,  $\rho$ .

Type `ndarray`

**bootstrapped\_beta**

Bootstrapped demand-side linear parameters,  $\beta$ .

Type `ndarray`

**bootstrapped\_gamma**

Bootstrapped supply-side linear parameters,  $\gamma$ .

Type `ndarray`

**bootstrapped\_prices**

Bootstrapped prices,  $p$ . If a supply side was not estimated, these are unchanged prices. Otherwise, they are equilibrium prices implied by each draw.

Type `ndarray`

**bootstrapped\_shares**

Bootstrapped market shares,  $s$ , implied by each draw.

Type `ndarray`

**bootstrapped\_delta**

Bootstrapped mean utility,  $\delta$ , implied by each draw.

Type `ndarray`

**computation\_time**

Number of seconds it took to compute the bootstrapped results.

Type `float`

**draws**

Number of bootstrap draws.

Type `int`

**fp\_converged**

Flags for convergence of the iteration routine used to compute equilibrium prices in each market. Rows are in the same order as `Problem.unique_market_ids` and column indices correspond to draws.

Type `ndarray`

**fp\_iterations**

Number of major iterations completed by the iteration routine used to compute equilibrium prices in each market for each draw. Rows are in the same order as `Problem.unique_market_ids` and column indices correspond to draws.

Type `ndarray`

**contraction\_evaluations**

Number of times the contraction used to compute equilibrium prices was evaluated in each market for each draw. Rows are in the same order as `Problem.unique_market_ids` and column indices correspond to draws.

Type `ndarray`

**Examples**

- *Tutorial*

This class has all of the same methods as `ProblemResults`, except for `ProblemResults.bootstrap()`, `ProblemResults.compute_optimal_instruments()`, and `ProblemResults.importance_sampling()`. It can also be converted into a dictionary.

---

<code>BootstrappedResults.to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
--	--

---

**5.6.2 pyblp.BootstrappedResults.to\_dict**

```
BootstrappedResults.to_dict(attributes=('bootstrapped_sigma', 'bootstrapped_pi', 'bootstrapped_rho', 'bootstrapped_beta', 'bootstrapped_gamma', 'bootstrapped_prices', 'bootstrapped_shares', 'bootstrapped_delta', 'computation_time', 'draws', 'fp_converged', 'fp_iterations', 'contraction_evaluations'))
```

Convert these results into a dictionary that maps attribute names to values.

Once converted to a dictionary, these results can be saved to a file with `pickle.dump()`.

**Parameters** `attributes` (*sequence of str, optional*) – Name of attributes that will be added to the dictionary. By default, all `BootstrappedResults` attributes are added except for `BootstrappedResults.problem_results`.

**Returns** Mapping from attribute names to values.

**Return type** `dict`

**Examples**

- *Tutorial*

**5.7 Optimal Instrument Results Class**

Optimal instrument computation returns the following results class.

## 5.7.1 pyblp.OptimalInstrumentResults

**class** `pyblp.OptimalInstrumentResults`

Results of optimal instrument computation.

The `OptimalInstrumentResults.to_problem()` method can be used to update the original `Problem` with the computed optimal instruments.

**problem\_results**

*ProblemResults* that was used to compute these optimal instrument results.

**Type** *ProblemResults*

**demand\_instruments**

Estimated optimal demand-side instruments for  $\theta$ , denoted  $Z_D^{\text{opt}}$ .

**Type** *ndarray*

**supply\_instruments**

Estimated optimal supply-side instruments for  $\theta$ , denoted  $Z_S^{\text{opt}}$ .

**Type** *ndarray*

**supply\_shifter\_formulation**

*Formulation* configuration for supply shifters that will by default be included in the full set of optimal demand-side instruments. This is only constructed if a supply side was estimated, and it can be changed in `OptimalInstrumentResults.to_problem()`. By default, this is the formulation for  $X_3^{\text{ex}}$  from `Problem` excluding any variables in the formulation for  $X_1^{\text{ex}}$ .

**Type** *Formulation or None*

**demand\_shifter\_formulation**

*Formulation* configuration for demand shifters that will by default be included in the full set of optimal supply-side instruments. This is only constructed if a supply side was estimated, and it can be changed in `OptimalInstrumentResults.to_problem()`. By default, this is the formulation for  $X_1^{\text{ex}}$  from `Problem` excluding any variables in the formulation for  $X_3^{\text{ex}}$ .

**Type** *Formulation or None*

**inverse\_covariance\_matrix**

Inverse of the sample covariance matrix of the estimated  $\xi$  and  $\omega$ , which is used to normalize the expected Jacobians. If a supply side was not estimated, this is simply the sample estimate of  $1/\sigma_\xi^2$ .

**Type** *ndarray*

**expected\_xi\_by\_theta\_jacobian**

Estimated  $E[\frac{\partial \xi}{\partial \theta} | Z]$ .

**Type** *ndarray*

**expected\_omega\_by\_theta\_jacobian**

Estimated  $E[\frac{\partial \omega}{\partial \theta} | Z]$ .

**Type** *ndarray*

**expected\_prices**

Vector of expected prices conditional on all exogenous variables,  $E[p | Z]$ , which may have been specified in `ProblemResults.compute_optimal_instruments()`.



**Type** *ndarray*

**expected\_shares**

Vector of expected market shares conditional on all exogenous variables,  $E[s | Z]$ .

**Type** *ndarray*

**computation\_time**

Number of seconds it took to compute optimal excluded instruments.

**Type** *float*

**draws**

Number of draws used to approximate the integral over the error term density.

**Type** *int*

**fp\_converged**

Flags for convergence of the iteration routine used to compute equilibrium prices in each market. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to draws.

**Type** *ndarray*

**fp\_iterations**

Number of major iterations completed by the iteration routine used to compute equilibrium prices in each market for each error term draw. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to draws.

**Type** *ndarray*

**contraction\_evaluations**

Number of times the contraction used to compute equilibrium prices was evaluated in each market for each error term draw. Rows are in the same order as *Problem.unique\_market\_ids* and column indices correspond to draws.

**Type** *ndarray*

**Examples**

- *Tutorial*

**Methods**

<code>to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
<code>to_problem([supply_shifter_formulation, ...])</code>	Re-create the problem with estimated feasible optimal instruments.

The results can be converted into a dictionary.

<code>OptimalInstrumentResults.to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
---	--

## 5.7.2 pyblp.OptimalInstrumentResults.to\_dict

`OptimalInstrumentResults.to_dict` (*attributes*=(*'demand\_instruments'*, *'supply\_instruments'*, *'inverse\_covariance\_matrix'*, *'expected\_xi\_by\_theta\_jacobian'*, *'expected\_omega\_by\_theta\_jacobian'*, *'expected\_prices'*, *'expected\_shares'*, *'computation\_time'*, *'draws'*, *'fp\_converged'*, *'fp\_iterations'*, *'contraction\_evaluations'*))

Convert these results into a dictionary that maps attribute names to values.

Once converted to a dictionary, these results can be saved to a file with `pickle.dump()`.

**Parameters** *attributes* (sequence of str, optional) – Name of attributes that will be added to the dictionary. By default, all `OptimalInstrumentResults` attributes are added except for `OptimalInstrumentResults.problem_results`, `OptimalInstrumentResults.supply_shifter_formulation`, and `OptimalInstrumentResults.demand_shifter_formulation`.

**Returns** Mapping from attribute names to values.

**Return type** *dict*

### Examples

- [Tutorial](#)

They can also be converted into a `Problem` with the following method.

<code>OptimalInstrumentResults.to_problem(...)</code>	Re-create the problem with estimated feasible optimal instruments.
---	--

## 5.7.3 pyblp.OptimalInstrumentResults.to\_problem

`OptimalInstrumentResults.to_problem` (*supply\_shifter\_formulation*=None, *demand\_shifter\_formulation*=None)

Re-create the problem with estimated feasible optimal instruments.

The re-created problem will be exactly the same, except that instruments will be replaced with estimated feasible optimal instruments.

**Note:** Most of the explanation here is only important if a supply side was estimated.

The optimal excluded demand-side instruments consist of the following:

1. Estimated optimal demand-side instruments for  $\theta$ ,  $Z_D^{\text{opt}}$ , excluding columns of instruments for any parameters on exogenous linear characteristics that were not concentrated out, but rather included in  $\theta$  by `Problem.solve()`.
2. Optimal instruments for any linear demand-side parameters on endogenous product characteristics,  $\alpha$ , which were concentrated out and hence not included in  $\theta$ . These optimal instruments are simply an integral of the endogenous product characteristics,  $X_1^{\text{en}}$ , over the joint density of  $\xi$  and  $\omega$ . It is only possible to concentrate out  $\alpha$  when there isn't a supply side, so the approximation of these optimal instruments is simply  $X_1^{\text{en}}$  evaluated at the constant vector of expected prices,  $E[p | Z]$ , specified in `ProblemResults.compute_optimal_instruments()`.

3. If a supply side was estimated, any supply shifters, which are by default formulated by `OptimalInstrumentResults.supply_shifter_formulation`: all characteristics in  $X_3^{\text{ex}}$  not in  $X_1^{\text{ex}}$ .

Similarly, if a supply side was estimated, the optimal excluded supply-side instruments consist of the following:

1. Estimated optimal supply-side instruments for  $\theta$ ,  $Z_S^{\text{opt}}$ , excluding columns of instruments for any parameters on exogenous linear characteristics that were not concentrated out, but rather included in  $\theta$  by `Problem.solve()`.
2. Optimal instruments for any linear supply-side parameters on endogenous product characteristics,  $\gamma^{\text{en}}$ , which were concentrated out and hence not included in  $\theta$ . This is only relevant if shares were included in the formulation for  $X_3$  in `Problem`. The corresponding optimal instruments are simply an integral of the endogenous product characteristics,  $X_3^{\text{en}}$ , over the joint density of  $\xi$  and  $\omega$ . The approximation of these optimal instruments is simply  $X_3^{\text{en}}$  evaluated at the market shares that arise under the constant vector of expected prices,  $E[p | Z]$ , specified in `ProblemResults.compute_optimal_instruments()`.
2. If a supply side was estimated, any demand shifters, which are by default formulated by `OptimalInstrumentResults.demand_shifter_formulation`: all characteristics in  $X_1^{\text{ex}}$  not in  $X_3^{\text{ex}}$ .

As usual, the excluded demand-side instruments will be supplemented with  $X_1^{\text{ex}}$  and the excluded supply-side instruments will be supplemented with  $X_3^{\text{ex}}$ . The same fixed effects configured in `Problem` will be absorbed.

**Warning:** If a supply side was estimated, the addition of supply- and demand-shifters may create collinearity issues. Make sure to check that shifters and other product characteristics are not collinear.

### Parameters

- **supply\_shifter\_formulation** (*Formulation, optional*) – *Formulation* configuration for supply shifters to be included in the set of optimal demand-side instruments. This is only used if a supply side was estimated. Intercepts will be ignored. By default, `OptimalInstrumentResults.supply_shifter_formulation` is used.
- **demand\_shifter\_formulation** (*Formulation, optional*) – *Formulation* configuration for demand shifters to be included in the set of optimal supply-side instruments. This is only used if a supply side was estimated. Intercepts will be ignored. By default, `OptimalInstrumentResults.demand_shifter_formulation` is used.

**Returns** `OptimalInstrumentProblem`, which is a `Problem` updated to use the estimated optimal instruments.

**Return type** `OptimalInstrumentProblem`

### Examples

- [Tutorial](#)

This method returns the following class, which behaves exactly like a `Problem`.

---

`OptimalInstrumentProblem`

A BLP problem updated with optimal excluded instruments.

---

## 5.7.4 pyblp.OptimalInstrumentProblem

**class** `pyblp.OptimalInstrumentProblem`

A BLP problem updated with optimal excluded instruments.

This class can be used exactly like *Problem*.

## 5.8 Importance Sampling Results Class

Importance sampling returns the following results class:

---

*ImportanceSamplingResults*

---

Results of importance sampling.

---

### 5.8.1 pyblp.ImportanceSamplingResults

**class** `pyblp.ImportanceSamplingResults`

Results of importance sampling.

Along with the sampled agents, these results also contain a number of useful importance sampling diagnostics from *Owen (2013)*.

The `ImportanceSamplingResults.to_problem()` method can be used to update the original *Problem* with the importance sampling agent data.

**problem\_results**

*ProblemResults* that was used to compute these importance sampling results.

**Type** *ProblemResults*

**sampled\_agents**

Importance sampling agent data structured as *Agents*. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *Agents*

**computation\_time**

Number of seconds it took to do importance sampling.

**Type** *float*

**draws**

Number of importance sampling draws in each market.

**Type** *int*

**diagnostic\_market\_ids**

Market IDs that correspond to the ordering of the following arrays of weight diagnostics.

**Type** *ndarray*

**weight\_sums**

Sum of weights in each market:  $\sum_i w_{it}$ . If importance sampling was successful, weights should not sum to numbers too far from one.

**Type** *ndarray*

**effective\_draws**

Effective sample sizes in each market:  $\frac{(\sum_i w_{it})^2}{\sum_i w_{it}^2}$ .

Type *ndarray*

#### **effective\_draws\_for\_variance**

Effective sample sizes for variance estimates in each market:  $\frac{(\sum_i w_{it}^2)^2}{\sum_i w_{it}^4}$ .

Type *ndarray*

#### **effective\_draws\_for\_skewness**

Effective sample sizes for gauging skewness in each market:  $\frac{(\sum_i w_{it}^2)^3}{(\sum_i w_{it}^3)^2}$ .

Type *ndarray*

### Examples

- [Tutorial](#)

### Methods

<code>to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
<code>to_problem()</code>	Re-create the problem with the agent data constructed from importance sampling.

The results can be converted into a dictionary.

<code>ImportanceSamplingResults.to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
--	--

## 5.8.2 `pyblp.ImportanceSamplingResults.to_dict`

`ImportanceSamplingResults.to_dict` (*attributes*=(*'sampled\_agents'*, *'computation\_time'*, *'draws'*, *'diagnostic\_market\_ids'*, *'weight\_sums'*, *'effective\_draws'*, *'effective\_draws\_for\_variance'*, *'effective\_draws\_for\_skewness'*))

Convert these results into a dictionary that maps attribute names to values.

Once converted to a dictionary, these results can be saved to a file with `pickle.dump()`.

**Parameters** *attributes* (*sequence of str, optional*) – Names of attributes that will be added to the dictionary. By default, all `ImportanceSamplingResults` attributes are added except for `ImportanceSamplingResults.problem_results`.

**Returns** Mapping from attribute names to values.

**Return type** *dict*

### Examples

- [Tutorial](#)

They can also be converted into a `Problem` with the following method.

---

*ImportanceSamplingResults.to\_problem()*Re-create the problem with the agent data constructed from importance sampling.

---

### 5.8.3 pyblp.ImportanceSamplingResults.to\_problem

`ImportanceSamplingResults.to_problem()`

Re-create the problem with the agent data constructed from importance sampling.

The re-created problem will be exactly the same, except *Problem.agents* will be replaced with *ImportanceSamplingResults.sampled\_agents*.**Returns** *ImportanceSamplingProblem*, which is a *Problem* updated to use agent data constructed from importance sampling.**Return type** *ImportanceSamplingProblem*

#### Examples

- *Tutorial*

This method returns the following class, which behaves exactly like a *Problem*.

---

*ImportanceSamplingProblem*A BLP problem updated after importance sampling.

---

### 5.8.4 pyblp.ImportanceSamplingProblem

**class** `pyblp.ImportanceSamplingProblem`

A BLP problem updated after importance sampling.

This class can be used exactly like *Problem*.

## 5.9 Simulation Class

The following class allows for evaluation of more complicated counterfactuals than is possible with *ProblemResults* methods, or for simulation of synthetic data from scratch.

---

*Simulation*(*product\_formulations*, ...[, ...])Simulation of data in BLP-type models.

---

### 5.9.1 pyblp.Simulation

**class** `pyblp.Simulation`(*product\_formulations*, *product\_data*, *beta*, *sigma=None*, *pi=None*, *gamma=None*, *rho=None*, *agent\_formulation=None*, *agent\_data=None*, *integration=None*, *xi=None*, *omega=None*, *xi\_variance=1*, *omega\_variance=1*, *correlation=0.9*, *distributions=None*, *epsilon\_scale=1.0*, *costs\_type='linear'*, *seed=None*)

Simulation of data in BLP-type models.

Any data left unspecified are simulated during initialization. Simulated prices and shares can be replaced by *Simulation.replace\_endogenous()* with equilibrium values that are consistent with true pa-

rameters. Less commonly, simulated exogenous variables can be replaced instead by `Simulation.replace_exogenous()`. To choose your own prices, refer to the first note in `Simulation.replace_endogenous()`. Simulations are typically used for two purposes:

1. Solving for equilibrium prices and shares under more complicated counterfactuals than is possible with `ProblemResults.compute_prices()` and `ProblemResults.compute_shares()`. For example, this class can be initialized with estimated parameters, structural errors, and marginal costs from a `ProblemResults()`, but with changed data (fewer products, new products, different characteristics, etc.) and `Simulation.replace_endogenous()` can be used to compute the corresponding prices and shares.
2. Simulation of BLP-type models from scratch. For example, a model with fixed true parameters can be simulated many times, converted into problems with `SimulationResults.to_problem()`, and solved with `Problem.solve()` to evaluate in a Monte Carlo study how well the true parameters can be recovered.

If data for variables (used to formulate product characteristics in  $X_1$ ,  $X_2$ , and  $X_3$ , as well as agent demographics,  $d$ , and endogenous prices and market shares  $p$  and  $s$ ) are not provided, the values for each unspecified variable are drawn independently from the standard uniform distribution. In each market  $t$ , market shares are divided by the number of products in the market  $J_t$ . Typically, `Simulation.replace_endogenous()` is used to replace prices and shares with equilibrium values that are consistent with true parameters.

If data for unobserved demand-and supply-side product characteristics,  $\xi$  and  $\omega$ , are not provided, they are by default drawn from a mean-zero bivariate normal distribution.

After variables are loaded or simulated, any unspecified integration nodes and weights,  $\nu$  and  $w$ , are constructed according to a specified `Integration` configuration.

### Parameters

- **product\_formulations** (*Formulation or sequence of Formulation*) – `Formulation` configuration or a sequence of up to three `Formulation` configurations for the matrix of demand-side linear product characteristics,  $X_1$ , for the matrix of demand-side nonlinear product characteristics,  $X_2$ , and for the matrix of supply-side characteristics,  $X_3$ , respectively. If the formulation for  $X_2$  is not specified or is `None`, the logit (or nested logit) model will be simulated.

The `shares` variable should not be included in the formulations for  $X_1$  or  $X_2$ . If `shares` is included in the formulation for  $X_3$ , care should be taken when solving for equilibrium prices in, for example, `Simulation.replace_endogenous()`, since this routine assumes that marginal costs remain constant.

The `prices` variable should not be included in the formulation for  $X_3$ , but it should be included in the formulation for  $X_1$  or  $X_2$  (or both). Variables that cannot be loaded from `product_data` will be drawn from independent standard uniform distributions. Unlike in `Problem`, fixed effect absorption is not supported during simulation.

**Warning:** Characteristics that involve prices,  $p$ , or shares,  $s$ , should always be formulated with the `prices` and `shares` variables, respectively. If another name is used, `Simulation` will not understand that the characteristic is endogenous. For example, to include a  $p^2$  characteristic, include `I(prices**2)` in a formula instead of manually constructing and including a `prices_squared` variable.

- **product\_data** (*structured array-like*) – Each row corresponds to a product. Markets can have differing numbers of products. The convenience function `build_id_data()` can be used to construct the following required ID data:
  - **market\_ids** : (*object*) - IDs that associate products with markets.

- **firm\_ids** : (*object*) - IDs that associate products with firms.

Custom ownership matrices can be specified as well:

- **ownership** : (*numeric, optional*) - Custom stacked  $J_t \times J_t$  ownership or product holding matrices,  $\mathcal{H}$ , for each market  $t$ , which can be built with `build_ownership()`. By default, standard ownership matrices are built only when they are needed to reduce memory usage. If specified, there should be as many columns as there are products in the market with the most products. Rightmost columns in markets with fewer products will be ignored.

---

**Note:** The `ownership` field can either be a matrix or can be broken up into multiple one-dimensional fields with column index suffixes that start at zero. For example, if there are three products in each market, a `ownership` field with three columns can be replaced by three one-dimensional fields: `ownership0`, `ownership1`, and `ownership2`.

---

To use certain types of micro moments, product IDs must be specified:

- **product\_ids** (*object, optional*) - IDs that identify individual products within markets. The IDs referenced by `DemographicExpectationMoment` or `DiversionProbabilityMoment` must be unique within the relevant markets.

To simulate a nested logit or random coefficients nested logit (RCNL) model, nesting groups must be specified:

- **nesting\_ids** (*object, optional*) - IDs that associate products with nesting groups. When these IDs are specified, `rho` must be specified as well.

Along with `market_ids`, `firm_ids`, `product_ids`, and `nesting_ids`, the names of any additional fields can typically be used as variables in `product_formulations`. However, there are a few variable names such as 'X1', which are reserved for use by `Products`.

- **beta** (*array-like*) – Vector of demand-side linear parameters,  $\beta$ . Elements correspond to columns in  $X_1$ , which is formulated by `product_formulations`.
- **sigma** (*array-like, optional*) – Lower-triangular Cholesky root of the covariance matrix for unobserved taste heterogeneity,  $\Sigma$ . Rows and columns correspond to columns in  $X_2$ , which is formulated by `product_formulations`. If  $X_2$  is not formulated, this should not be specified, since the logit model will be simulated.
- **pi** (*array-like, optional*) – Parameters that measure how agent tastes vary with demographics,  $\Pi$ . Rows correspond to the same product characteristics as in `sigma`. Columns correspond to columns in  $d$ , which is formulated by `agent_formulation`. If  $d$  is not formulated, this should not be specified.
- **gamma** (*array-like, optional*) – Vector of supply-side linear parameters,  $\gamma$ . Elements correspond to columns in  $X_3$ , which is formulated by `product_formulations`. If  $X_3$  is not formulated, this should not be specified.
- **rho** (*array-like, optional*) – Parameters that measure within nesting group correlation,  $\rho$ . If this is a scalar, it corresponds to all groups defined by the `nesting_ids` field of `product_data`. If this is a vector, it must have  $H$  elements, one for each nesting group. Elements correspond to group IDs in the sorted order of `Simulation.unique_nesting_ids`. If nesting IDs are not specified, this should not be specified either.
- **agent\_formulation** (*Formulation, optional*) – `Formulation` configuration for the matrix of observed agent characteristics called demographics,  $d$ , which will only be included



in the model if this formulation is specified. Any variables that cannot be loaded from `agent_data` will be drawn from independent standard uniform distributions.

- **agent\_data** (*structured array-like, optional*) – Each row corresponds to an agent. Markets can have differing numbers of agents. Since simulated agents are only used if there are demand-side nonlinear product characteristics, agent data should only be specified if  $X_2$  is formulated in `product_formulations`. If agent data are specified, market IDs are required:
  - **market\_ids** : (*object, optional*) - IDs that associate agents with markets. The set of distinct IDs should be the same as the set in `product_data`. If `integration` is specified, there must be at least as many rows in each market as the number of nodes and weights that are built for the market.

If `integration` is not specified, the following fields are required:

- **weights** : (*numeric, optional*) - Integration weights,  $w$ , for integration over agent choice probabilities.
- **nodes** : (*numeric, optional*) - Unobserved agent characteristics called integration nodes,  $\nu$ . If there are more than  $K_2$  columns (the number of demand-side nonlinear product characteristics), only the first  $K_2$  will be used. If any columns of `sigma` are fixed at zero, only the first few columns of these nodes will be used.

The convenience function `build_integration()` can be useful when constructing custom nodes and weights.

---

**Note:** If `nodes` has multiple columns, it can be specified as a matrix or broken up into multiple one-dimensional fields with column index suffixes that start at zero. For example, if there are three columns of nodes, a `nodes` field with three columns can be replaced by three one-dimensional fields: `nodes0`, `nodes1`, and `nodes2`.

---

Along with `market_ids`, the names of any additional fields can typically be used as variables in `agent_formulation`. The exception is the name 'demographics', which is reserved for use by `Agents`.

- **integration** (*Integration, optional*) – `Integration` configuration for how to build nodes and weights for integration over agent choice probabilities, which will replace any `nodes` and `weights` fields in `agent_data`. This configuration is required if `nodes` and `weights` in `agent_data` are not specified. It should not be specified if  $X_2$  is not formulated in `product_formulations`.

If this configuration is specified,  $K_2$  columns of nodes (the number of demand-side nonlinear product characteristics) will be built. However, if `sigma` is left unspecified or is specified with columns fixed at zero, fewer columns will be used.

- **xi** (*array-like, optional*) – Unobserved demand-side product characteristics,  $\xi$ . By default, if  $X_3$  is formulated, each pair of unobserved characteristics in this vector and  $\omega$  is drawn from a mean-zero bivariate normal distribution. This must be specified if  $X_3$  is not formulated or if `omega` is specified.
- **omega** (*array-like, optional*) – Unobserved supply-side product characteristics,  $\omega$ . By default, if  $X_3$  is formulated, each pair of unobserved characteristics in this vector and  $\xi$  is drawn from a mean-zero bivariate normal distribution. This must be specified if  $X_3$  is formulated and `xi` is specified. It is ignored if  $X_3$  is not formulated.
- **xi\_variance** (*float, optional*) – Variance of  $\xi$ . The default value is 1.0. This is ignored if `xi` or `omega` is specified.

- **omega\_variance** (*float, optional*) – Variance of  $\omega$ . The default value is 1.0. This is ignored if `xi` or `omega` is specified.
- **correlation** (*float, optional*) – Correlation between  $\xi$  and  $\omega$ . The default value is 0.9. This is ignored if `xi` or `omega` is specified.
- **distributions** (*sequence of str, optional*) – Random coefficient distributions. By default, random coefficients in (3.3) are assumed to be normally distributed. Non-default distributions can be specified with a list of the following supported strings:
  - 'normal' (default) - The random coefficient is assumed to be normal.
  - 'lognormal' - The random coefficient is assumed to be lognormal. The coefficient's column in (3.3) is exponentiated before being pre-multiplied by  $X_2$ .

The list should have as many strings as there are columns in  $X_2$ . Each string determines the distribution of the random coefficient on the corresponding product characteristic in  $X_2$ .

A typical example of a lognormal coefficient is one on prices. Implementing this typically involves having a  $I(-prices)$  in the formulation for  $X_2$ , and instead of including `prices` in  $X_1$ , including a 1 in the `agent_formulation`. Then the corresponding coefficient in  $\Pi$  will serve as the mean parameter for the lognormal random coefficient on negative prices,  $-p_{jt}$ .

- **epsilon\_scale** (*float, optional*) – Factor by which the Type I Extreme Value idiosyncratic preference term,  $\epsilon_{ijt}$ , is scaled. By default,  $\epsilon_{ijt}$  is not scaled. The typical use of this parameter is to approximate the pure characteristics model of *Berry and Pakes (2007)* by choosing a value smaller than 1.0. As this scaling factor approaches zero, the model approaches the pure characteristics model in which there is no idiosyncratic preference term.

For more information about choosing this parameter and estimating models where it is smaller than 1.0, refer to the same argument in `Problem.solve()`. In some situations, it may be easier to solve simulations with small epsilon scaling factors by using `Simulation.replace_exogenous()` rather than `Simulation.replace_endogenous()`.

- **costs\_type** (*str, optional*) – Specification of the marginal cost function  $\tilde{c} = f(c)$  in (3.9). The following specifications are supported:
  - 'linear' (default) - Linear specification:  $\tilde{c} = c$ .
  - 'log' - Log-linear specification:  $\tilde{c} = \log c$ .
- **seed** (*int, optional*) – Passed to `numpy.random.RandomState` to seed the random number generator before data are simulated. By default, a seed is not passed to the random number generator.

#### **product\_formulations**

*Formulation* configurations for  $X_1$ ,  $X_2$ , and  $X_3$ , respectively.

Type *tuple*

#### **agent\_formulation**

*Formulation* configuration for  $d$ .

Type *tuple*

#### **product\_data**

Synthetic product data that were loaded or simulated during initialization. Typically, `Simulation.replace_endogenous()` is used to replace prices and shares with equilibrium values that are consistent with true parameters. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *recarray*

**agent\_data**

Synthetic agent data that were loaded or simulated during initialization. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *recarray*

**integration**

*Integration* configuration for how any nodes and weights were built during initialization.

**Type** *Integration*

**products**

Product data structured as *Products*, which consists of data taken from *Simulation.product\_data* along with matrices build according to *Simulation.product\_formulations*. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *Products*

**agents**

Agent data structured as *Agents*, which consists of data taken from *Simulation.agent\_data* or built by *Simulation.integration* along with any demographics formulated by *Simulation.agent\_formulation*. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *Agents*

**unique\_market\_ids**

Unique market IDs in product and agent data.

**Type** *ndarray*

**unique\_firm\_ids**

Unique firm IDs in product data.

**Type** *ndarray*

**unique\_product\_ids**

Unique product IDs in product data.

**Type** *ndarray*

**unique\_nesting\_ids**

Unique nesting IDs in product data.

**Type** *ndarray*

**beta**

Demand-side linear parameters,  $\beta$ .

**Type** *ndarray*

**sigma**

Cholesky root of the covariance matrix for unobserved taste heterogeneity,  $\Sigma$ .

**Type** *ndarray*

**gamma**

Supply-side linear parameters,  $\gamma$ .

**Type** *ndarray*

**pi**

Parameters that measures how agent tastes vary with demographics,  $\Pi$ .

**Type** *ndarray*

**rho**

Parameters that measure within nesting group correlation,  $\rho$ .

**Type** *ndarray*

**xi**

Unobserved demand-side product characteristics,  $\xi$ .

**Type** *ndarray*

**omega**

Unobserved supply-side product characteristics,  $\omega$ .

**Type** *ndarray*

**distributions**

Random coefficient distributions.

**Type** *list of str*

**epsilon\_scale**

Factor by which the Type I Extreme Value idiosyncratic preference term,  $\epsilon_{ijt}$ , is scaled.

**Type** *float*

**costs\_type**

Functional form of the marginal cost function  $\tilde{c} = f(c)$ .

**Type** *str*

**T**

Number of markets,  $T$ .

**Type** *int*

**N**

Number of products across all markets,  $N$ .

**Type** *int*

**F**

Number of firms across all markets,  $F$ .

**Type** *int*

**I**

Number of agents across all markets,  $I$ .

**Type** *int*

**K1**

Number of demand-side linear product characteristics,  $K_1$ .

**Type** *int*

**K2**

Number of demand-side nonlinear product characteristics,  $K_2$ .

**Type** *int*

**K3**

Number of supply-side characteristics,  $K_3$ .

**Type** *int*

- D**  
Number of demographic variables,  $D$ .  
**Type** *int*
- MD**  
Number of demand-side instruments,  $M_D$ , which is always zero because instruments are added or constructed in `SimulationResults.to_problem()`.  
**Type** *int*
- MS**  
Number of supply-side instruments,  $M_S$ , which is always zero because instruments are added or constructed in `SimulationResults.to_problem()`.  
**Type** *int*
- ED**  
Number of absorbed dimensions of demand-side fixed effects,  $E_D$ , which is always zero because simulations do not support fixed effect absorption.  
**Type** *int*
- ES**  
Number of absorbed dimensions of supply-side fixed effects,  $E_S$ , which is always zero because simulations do not support fixed effect absorption.  
**Type** *int*
- H**  
Number of nesting groups,  $H$ .  
**Type** *int*

## Examples

- [Tutorial](#)

## Methods

<code>replace_endogenous([costs, prices, ...])</code>	Replace simulated prices and market shares with equilibrium values that are consistent with true parameters.
<code>replace_exogenous(X1_name[, X3_name, delta, ...])</code>	Replace exogenous product characteristics with values that are consistent with true parameters.

Once initialized, the following method replaces prices and shares with equilibrium values that are consistent with true parameters.

<code>Simulation.replace_endogenous([costs, ...])</code>	Replace simulated prices and market shares with equilibrium values that are consistent with true parameters.
--	--

## 5.9.2 pyblp.Simulation.replace\_endogenous

`Simulation.replace_endogenous` (*costs=None, prices=None, iteration=None, error\_behavior='raise'*)

Replace simulated prices and market shares with equilibrium values that are consistent with true parameters.

This method is the standard way of solving the simulation. Prices and market shares are computed in each market by iterating over the  $\zeta$ -markup contraction in (3.46):

$$p \leftarrow c + \zeta(p). \quad (5.35)$$

---

**Note:** To not replace prices, pass the desired prices to `prices` and use an `Iteration` configuration with `method='return'`. This just uses the iteration “routine” that simply returns the the starting values, which are `prices`.

Using this same fake iteration routine and not setting prices will result in a simulation under perfect (instead of Bertrand) competition because the default starting values for the iteration routine are marginal costs.

---



---

**Note:** This method supports `parallel()` processing. If multiprocessing is used, market-by-market computation of prices and shares will be distributed among the processes.

---

### Parameters

- **costs** (*array-like, optional*) – Marginal costs,  $c$ . By default,  $c = X_3\gamma + w$  if `costs_type` was 'linear' in `Simulation` (the default), and the exponential of this if it was 'log'. Marginal costs must be specified if  $X_3$  was not formulated in `Simulation`. If marginal costs depend on prices through market shares, they will be updated to reflect different prices during each iteration of the routine.
- **prices** (*array-like, optional*) – Prices at which the fixed point iteration routine will start. By default, `costs`, are used as starting values.
- **iteration** (*Iteration, optional*) – `Iteration` configuration for how to solve the fixed point problem. By default, `Iteration('simple', {'atol': 1e-12})` is used. Analytic Jacobians are not supported for solving this system.
- **error\_behavior** (*str, optional*) – How to handle errors when computing prices and shares. For example, the fixed point routine may not converge if the effects of nonlinear parameters on price overwhelm the linear parameter on price, which should be sufficiently negative. The following behaviors are supported:
  - 'raise' (default) - Raise an exception.
  - 'warn' - Use the last computed prices and shares. If the fixed point routine fails to converge, these are the last prices and shares computed by the routine. If there are other issues, these are the starting prices and their associated shares.

**Returns** `SimulationResults` of the solved simulation.

**Return type** `SimulationResults`

### Examples

- [Tutorial](#)

A less common way to solve the simulation is to assume simulated prices and shares represent an equilibrium and to replace exogenous variables instead.

---

<code>Simulation.replace_exogenous(X1_name[, ...])</code>	Replace exogenous product characteristics with values that are consistent with true parameters.
---	---

---

### 5.9.3 pyblp.Simulation.replace\_exogenous

`Simulation.replace_exogenous(X1_name, X3_name=None, delta=None, iteration=None, fp_type='safe_linear', shares_bounds=(1e-300, None), error_behavior='raise')`

Replace exogenous product characteristics with values that are consistent with true parameters.

This method implements a less common way of solving the simulation. It may be preferable to `Simulation.replace_endogenous()` when for some reason it is desirable to retain the prices and market shares from `Simulation`, which are assumed to be in equilibrium. For example, it can be helpful when approximating the pure characteristics model of *Berry and Pakes (2007)* by setting a small `epsilon_scale` value in `Simulation`.

For this method of solving the simulation to be used, there must be an exogenous product characteristic  $v$  that shows up only in  $X_1^{\text{ex}}$ , and if there is a supply side, another product characteristic  $w$  that shows up only in  $X_3^{\text{ex}}$ . These characteristics will be replaced with values that are consistent with true parameters.

First, the mean utility  $\delta$  is computed in each market by iterating over the contraction in (3.13) and  $(\delta - \xi - X_1\beta)\beta_v^{-1}$  is added to the  $v$  from `Simulation`. Here,  $\beta_v$  is the linear parameter in  $\beta$  on  $v$ .

With a supply side, the marginal cost function  $\tilde{c}$  is computed according to (3.7) and (3.9) and  $(\tilde{c} - \omega - X_3\gamma)\gamma_w^{-1}$  is added to the  $w$  from `Simulation`. Here,  $\gamma_w$  is the linear parameter in  $\gamma$  on  $w$ .

---

**Note:** This method supports `parallel()` processing. If multiprocessing is used, market-by-market computation of prices and shares will be distributed among the processes.

---

#### Parameters

- **X1\_name** (*str*) – The name of the variable  $v$  in  $X_1^{\text{ex}}$  that will be replaced. It should show up only once in the formulation for  $X_1$  from `Simulation` and it should not be transformed in any way.
- **X3\_name** (*str, optional*) – The name of the variable  $w$  in  $X_3^{\text{ex}}$  that will be replaced. It should show up only once in the formulation for  $X_3$  from `Simulation` and it should not be transformed in any way. This will only be used if there is a supply side.
- **delta** (*array-like, optional*) – Initial values for the mean utility,  $\delta$ , which the fixed point iteration routine will start at. By default, the solution to the logit model in (3.40) is used. If there is a nesting structure, solution to the nested logit model in (3.41) under the initial `rho` is used instead.
- **iteration** (*Iteration, optional*) – `Iteration` configuration for how to solve the fixed point problem used to compute  $\delta$  in each market. This configuration is only relevant if there are nonlinear parameters, since  $\delta$  can be estimated analytically in the logit model. By default, `Iteration('squarem', {'atol': 1e-14})` is used. For more information, refer to the same argument in `Problem.solve()`.
- **fp\_type** (*str, optional*) – Configuration for the type of contraction mapping used to compute  $\delta$ . For information about the different types, refer to the same argument in `Problem.solve()`.

- **shares\_bounds** (*tuple, optional*) – Configuration for  $s_{jt}(\delta, \theta)$  bounds of the form (lb, ub), in which both lb and ub are floats or None. By default, simulated shares are bounded from below by  $1e-300$ . This is only relevant if `fp_type` is 'safe\_linear' or 'linear'. Bounding shares in the contraction does nothing with a nonlinear fixed point. For more information, refer to `Problem.solve()`.
- **error\_behavior** (*str, optional*) – How to handle errors when computing  $\delta$  and  $\tilde{c}$ . The following behaviors are supported:
  - 'raise' (default) - Raise an exception.
  - 'warn' - Use the last computed  $\delta$  and  $\tilde{c}$ . If the fixed point routine fails to converge, these are the last  $\delta$  and the associated  $\tilde{c}$  by the routine. If there are other issues, these are the starting  $\delta$  values and their associated  $\tilde{c}$ .

**Returns** `SimulationResults` of the solved simulation.

**Return type** `SimulationResults`

### Examples

- *Tutorial*

## 5.10 Simulation Results Class

Solved simulations return the following results class.

---

<code>SimulationResults</code>	Results of a solved simulation of synthetic BLP data.
--------------------------------	---

---

### 5.10.1 pyblp.SimulationResults

**class** `pyblp.SimulationResults`

Results of a solved simulation of synthetic BLP data.

The `SimulationResults.to_problem()` method can be used to convert the full set of simulated data (along with some basic default instruments) and configured information into a `Problem`.

**simulation**

*Simulation* that created these results.

**Type** *Simulation*

**product\_data**

Simulated `Simulation.product_data` with product characteristics replaced so as to be consistent with the true parameters. If `Simulation.replace_endogenous()` was used to create these results, prices and market shares were replaced. If `Simulation.replace_exogenous()` was used, exogenous characteristics were replaced instead. The `data_to_dict()` function can be used to convert this into a more usable data type.

**Type** *recarray*

**delta**

Simulated mean utility,  $\delta$ .

**Type** *ndarray*



**costs**

Simulated marginal costs,  $c$ .

Type *ndarray*

**computation\_time**

Number of seconds it took to compute prices and market shares.

Type *float*

**fp\_converged**

Flags for convergence of the iteration routine used to compute prices or  $\delta$  (depending on the method used to create these results) in each market. Flags are in the same order as *Simulation.unique\_market\_ids*.

Type *ndarray*

**fp\_iterations**

Number of major iterations completed by the iteration routine used to compute prices or  $\delta$  in each market. Counts are in the same order as *Simulation.unique\_market\_ids*.

Type *ndarray*

**contraction\_evaluations**

Number of times the contraction used to compute prices or  $\delta$  was evaluated in each market. Counts are in the same order as *Simulation.unique\_market\_ids*.

Type *ndarray*

**Examples**

- *Tutorial*

**Methods**

<code>compute_micro_values(micro_moments)</code>	Compute simulated micro moment values $v_{mt}$ .
<code>to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
<code>to_problem([product_formulations, ...])</code>	Convert the solved simulation into a problem.

The results can be converted into a dictionary.

<code>SimulationResults.to_dict([attributes])</code>	Convert these results into a dictionary that maps attribute names to values.
--	--

**5.10.2 pyblp.SimulationResults.to\_dict**

`SimulationResults.to_dict(attributes=('product_data', 'computation_time', 'fp_converged', 'fp_iterations', 'contraction_evaluations'))`

Convert these results into a dictionary that maps attribute names to values.

Once converted to a dictionary, these results can be saved to a file with `pickle.dump()`.

**Parameters attributes** (*sequence of str, optional*) – Name of attributes that will be added to the dictionary. By default, all *SimulationResults* attributes are added except for *SimulationResults.simulation*.

**Returns** Mapping from attribute names to values.

**Return type** *dict*

### Examples

- *Tutorial*

They can also be converted into a *Problem* with the following method.

---

<i>SimulationResults.to_problem</i> ([...])	Convert the solved simulation into a problem.
---	---

---

### 5.10.3 pyblp.SimulationResults.to\_problem

`SimulationResults.to_problem` (*product\_formulations=None*, *product\_data=None*,  
*agent\_formulation=None*, *agent\_data=None*, *integration=None*,  
*distributions=None*, *epsilon\_scale=None*, *costs\_type=None*,  
*add\_exogenous=True*)

Convert the solved simulation into a problem.

Arguments are the same as those of *Problem*. By default, the structure of the problem will be the same as that of the solved simulation.

By default, some simple “sums of characteristics” BLP instruments are constructed. Demand-side instruments are constructed by *build\_blp\_instruments*() from variables in  $X_1^{extex}$ , along with any supply shifters (variables in  $X_3^{extex}$  but not  $X_1^{extex}$ ). Supply side instruments are constructed from variables in  $X_3^{extex}$ , along with any demand shifters (variables in  $X_1^{extex}$  but not  $X_3^{extex}$ ). Instruments will also be constructed from columns of ones if there is variation in  $J_t$ , the number of products per market. Any constant columns will be dropped. For example, if each firm owns exactly one product in each market, the “rival” columns of instruments will be zero and hence dropped.

---

**Note:** These excluded instruments are constructed only for convenience. Especially for more complicated problems, they should be replaced with better instruments.

---

#### Parameters

- **product\_formulations** (*Formulation or sequence of Formulation, optional*) – By default, *Simulation.product\_formulations*.
- **product\_data** (*structured array-like, optional*) – By default, *SimulationResults.product\_data* with excluded instruments.
- **agent\_formulation** (*Formulation, optional*) – By default, *Simulation.agent\_formulation*.
- **agent\_data** (*structured array-like, optional*) – By default, *Simulation.agent\_data*.
- **integration** (*Integration, optional*) – By default, this is unspecified.
- **distributions** (*sequence of str, optional*) – By default, *Simulation.distributions*.
- **epsilon\_scale** (*float, optional*) – By default, *Simulation.epsilon\_scale*.
- **costs\_type** (*str, optional*) – By default, *Simulation.costs\_type*.

- **add\_exogenous** (*bool, optional*) – By default, True.

**Returns** A BLP problem.

**Return type** *Problem*

### Examples

- *Tutorial*

Simulation results can also be used to compute micro moment values.

---

<i>SimulationResults.</i>	Compute simulated micro moment values $v_{mt}$ .
<i>compute_micro_values(...)</i>	

---

## 5.10.4 pyblp.SimulationResults.compute\_micro\_values

`SimulationResults.compute_micro_values` (*micro\_moments*)

Compute simulated micro moment values  $v_{mt}$ .

**Parameters** **micro\_moments** (*sequence of Moment*) – Configurations for the micro moments. For a list of supported moments, refer to *Micro Moment Classes*. Since only  $v_{mt}$ 's are computed, the values given when initializing the moments will be ignored.

**Returns** Micro moment values,  $v_{mt}$ . Rows are in the same order as *Simulation.unique\_market\_ids*. Columns are in the same order as *micro\_moments*. If a micro moment is not computed in one or more markets, the associated values will be `numpy.nan`.

**Return type** *ndarray*

### Examples

- *Tutorial*

## 5.11 Structured Data Classes

Product and agent data that are passed or constructed by *Problem* and *Simulation* are structured internally into classes with field names that more closely resemble BLP notation. Although these structured data classes are not directly constructable, they can be accessed with *Problem* and *Simulation* class attributes. It can be helpful to compare these structured data classes with the data or configurations used to create them.

---

<i>Products</i>	Product data structured as a record array.
<i>Agents</i>	Agent data structured as a record array.

---

### 5.11.1 pyblp.Products

**class** `pyblp.Products`

Product data structured as a record array.

Attributes in addition to the ones below are the variables underlying  $X_1$ ,  $X_2$ , and  $X_3$ .

**market\_ids**

IDs that associate products with markets.

Type *ndarray*

**firm\_ids**

IDs that associate products with firms.

Type *ndarray*

**demand\_ids**

IDs used to create demand-side fixed effects.

Type *ndarray*

**supply\_ids**

IDs used to create supply-side fixed effects.

Type *ndarray*

**nesting\_ids**

IDs that associate products with nesting groups.

Type *ndarray*

**product\_ids**

IDs that identify individual products within markets.

Type *ndarray*

**clustering\_ids**

IDs used to compute clustered standard errors.

Type *ndarray*

**ownership**

Stacked  $J_t \times J_t$  ownership or product holding matrices,  $\mathcal{H}$ , for each market  $t$ .

Type *ndarray*

**shares**

Marketshares,  $s$ .

Type *ndarray*

**prices**

Product prices,  $p$ .

Type *ndarray*

**ZD**

Full set of demand-side instruments,  $Z_D$ , which typically consists of excluded demand-side instruments and  $X_1^{\text{ex}}$ .

Type *ndarray*

**ZS**

Full set of supply-side instruments,  $Z_S$ , which typically consists of excluded supply-side instruments and  $X_3^{\text{ex}}$ .

Type *ndarray*

**X1**

Demand-side linear product characteristics,  $X_1$ .

Type *ndarray*

**x2**  
Demand-side nonlinear product characteristics,  $X_2$ .

**Type** *ndarray*

**x3**  
Supply-side product characteristics,  $X_3$ .

**Type** *ndarray*

## 5.11.2 pyblp.Agents

**class** `pyblp.Agents`

Agent data structured as a record array.

**market\_ids**

IDs that associate agents with markets.

**Type** *ndarray*

**weights**

Integration weights,  $w$ .

**Type** *ndarray*

**nodes**

Unobserved agent characteristics called integration nodes,  $\nu$ .

**Type** *ndarray*

**demographics**

Observed agent characteristics,  $d$ .

**Type** *ndarray*

## 5.12 Multiprocessing

A context manager can be used to enable parallel processing for methods that perform market-by-market computation.

---

`parallel(processes)`

Context manager used for parallel processing in a `with` statement context.

---

### 5.12.1 pyblp.parallel

`pyblp.parallel` (*processes*)

Context manager used for parallel processing in a `with` statement context.

This manager creates a context in which a pool of Python processes will be used by any method that requires market-by-market computation. These methods will distribute their work among the processes. After the context created by the `with` statement ends, all worker processes in the pool will be terminated. Outside of this context, such methods will not use multiprocessing.

Importantly, multiprocessing will only improve speed if gains from parallelization outweigh overhead from serializing and passing data between processes. For example, if computation for a single market is very fast and there is a lot of data in each market that must be serialized and passed between processes, using multiprocessing may reduce overall speed.

**Parameters** `processes` (*int*) – Number of Python processes that will be created and used by any method that supports parallel processing.

### Examples

---

The [online version](#) of the following section may be easier to read.

---

## Parallel Processing Example

```
import pyblp
import pandas as pd

pyblp.options.digits = 2
pyblp.options.verbose = False
pyblp.__version__

'0.10.1'
```

In this example, we'll use parallel processing to compute elasticities market-by-market for a simple Logit problem configured with some of the fake cereal data from *Nevo (2000)*.

```
product_data = pd.read_csv(pyblp.data.NEVO_PRODUCTS_LOCATION)
formulation = pyblp.Formulation('0 + prices', absorb='C(product_ids)')
problem = pyblp.Problem(formulation, product_data)
results = problem.solve()
results
```

Problem Results Summary:

```
=====
GMM   Objective   Clipped   Weighting Matrix
Step  Value        Shares    Condition Number
-----
  2    +1.9E+02     0         +5.7E+07
=====
```

Cumulative Statistics:

```
=====
Computation   Objective
Time          Evaluations
-----
00:00:00      2
=====
```

Beta Estimates (Robust SEs in Parentheses):

(continues on next page)

(continued from previous page)

```
=====  
prices  
-----  
-3.0E+01  
(+1.0E+00)  
=====
```

```
pyblp.options.verbose = True  
with pyblp.parallel(2):  
    elasticities = results.compute_elasticities()
```

```
Starting a pool of 2 processes ...  
Started the process pool after 00:00:00.  
Computing elasticities with respect to prices ...  
Finished after 00:00:04.
```

```
Terminating the pool of 2 processes ...  
Terminated the process pool after 00:00:00.
```

Solving a Logit problem does not require market-by-market computation, so parallelization does not change its estimation procedure. Although elasticity computation does happen market-by-market, this problem is very small, so in this small example there are no gains from parallelization.

If the problem were much larger, running *Problem.solve* and *ProblemResults.compute\_elasticities* under the `with` statement could substantially speed up estimation and elasticity computation.



## 5.13 Options and Example Data

In addition to classes and functions, there are also two modules that can be used to configure global package options and locate example data that comes with the package.

<code>options</code>	Global options.
<code>data</code>	Locations of example data that are included in the package for convenience.

### 5.13.1 `pyblp.options`

Global options.

`pyblp.options.digits`

Number of digits displayed by status updates. The default number of digits is 7. The number of digits can be changed to, for example, 2, with `pyblp.options.digits = 2`.

**Type** *int*

`pyblp.options.verbose`

Whether to output status updates. By default, verbosity is turned on. Verbosity can be turned off with `pyblp.options.verbose = False`.

**Type** *bool*

`pyblp.options.verbose_tracebacks`

Whether to include full tracebacks in error messages. By default, full tracebacks are turned off. These can be useful when attempting to find the source of an error message. Tracebacks can be turned on with `pyblp.options.verbose_tracebacks = True`.

**Type** *bool*

`pyblp.options.verbose_output`

Function used to output status updates. The default function is simply `print`. The function can be changed, for example, to include an indicator that statuses are from this package, with `pyblp.verbose_output = lambda x: print(f"pyblp: {x}")`.

**Type** *callable*

`pyblp.options.flush_output`

Whether to call `sys.stdout.flush()` after outputting a status update. By default, output is not flushed to standard output. To force standard output flushes after every status update, set `pyblp.options.flush_output = True`. This may be particularly desirable for R users who are calling PyBLP from `reticulate` (<https://github.com/rstudio/reticulate>), since standard output is typically not automatically flushed to the screen in this environment. If PyBLP is imported as `pyblp`, this setting can be enabled in R with `pyblp$options$flush_output <- TRUE`.

**Type** *bool*

`pyblp.options.dtype`

The data type used for internal calculations, which is by default `numpy.float64`. The other recommended option is `numpy.longdouble`, which is the only extended precision floating point type currently supported by NumPy. Although this data type will be used internally, `numpy.float64` will be used when passing arrays to optimization and fixed point routines, which may not support extended precision. The library underlying `scipy.linalg`, which is used for matrix inversion, may also use `numpy.float64`.

One instance in which extended precision can be helpful in the BLP problem is when there are a large number of near zero choice probabilities with small integration weights, which, under standard precision are called zeros when in aggregate they are nonzero. For example, *Skrainka (2012)* finds that using long doubles is sufficient to solve many utility floating point problems.

The precision of `numpy.longdouble` depends on the platform on which NumPy is installed. If the platform in use does not support extended precision, using `numpy.longdouble` may lead to unreliable results. For example, on Windows, NumPy is usually compiled such that `numpy.longdouble` often behaves like `numpy.float64`. Precisions can be compared with `numpy.finfo` by running `numpy.finfo(numpy.float64)` and `numpy.finfo(numpy.longdouble)`. For more information, refer to [this discussion](#).

If extended precisions is supported, the data type can be switched with `pyblp.options.dtype = numpy.longdouble`. On Windows, it is often easier to install Linux in a virtual machine than it is to build NumPy from source with a non-standard compiler.

#### Type *dtype*

##### `pyblp.options.finite_differences_epsilon`

Perturbation  $\epsilon$  used to numerically approximate derivatives with central finite differences:

$$f'(x) = \frac{f(x + \epsilon/2) - f(x - \epsilon/2)}{\epsilon}. \quad (5.36)$$

By default, this is the square root of the machine epsilon: `numpy.sqrt(numpy.finfo(options.dtype).eps)`. The typical example where this is used is when computing the Hessian, but it may also be used to compute Jacobians required for standard errors when analytic gradients are disabled.

#### Type *float*

##### `pyblp.options.pseudo_inverses`

Whether to compute Moore-Penrose pseudo-inverses of matrices with `scipy.linalg.pinv()` instead of their classic inverses with `scipy.linalg.inv()`. This is by default `True`, so pseudo-inverses will be used. Up to small numerical differences, the pseudo-inverse is identical to the classic inverse for invertible matrices. Using the pseudo-inverse by default can help alleviate problems from, for example, near-singular weighting matrices.

To always attempt to compute classic inverses first, set `pyblp.options.pseudo_inverses = False`. If a classic inverse cannot be computed, an error will be displayed, and a pseudo-inverse may be computed instead.

#### Type *bool*

##### `pyblp.options.collinear_atol`

Absolute tolerance for detecting collinear columns in each matrix of product characteristics and instruments:  $X_1, X_2, X_3, Z_D$ , and  $Z_S$ .

Each matrix is decomposed into a  $QR$  decomposition and an error is raised for any column whose diagonal element in  $R$  has a magnitude less than `collinear_atol + collinear_rtol * sd` where `sd` is the column's standard deviation.

The default absolute tolerance is  $1e-14$ . To disable collinearity checks, set `pyblp.options.collinear_atol = pyblp.options.collinear_rtol = 0`.

#### Type *float*

##### `pyblp.options.weights_tol`

Tolerance for detecting integration weights that do not sum to one in each market, which is by default  $1e-10$ . In most setups weights should essentially sum to one, but for example with importance sampling they may be slightly different. Warnings can be disabled by setting this to `numpy.inf`.

#### Type *float*

`pyblp.options.collinear_rtol`

Relative tolerance for detecting collinear columns, which is by default also  $1e-14$ .

**Type** *float*

`pyblp.options.psd_atol`

Absolute tolerance for detecting non-positive semidefinite matrices. For example, this check is applied to any custom weighting matrix,  $W$ .

Singular value decomposition factorizes the matrix into  $U\Sigma V$  and an error is raised if any element in the original matrix differs in absolute value from  $V'\Sigma V$  by more than `psd_atol + psd_rtol * abs` where `abs` is the element's absolute value.

The default tolerance is  $1e-8$ . To disable positive semidefinite checks, set `pyblp.options.psd_atol = pyblp.options.psd_rtol = numpy.inf`.

**Type** *float*

`pyblp.options.psd_rtol`

Relative tolerance for detecting non-positive definite matrices, which is by default also  $1e-8$ .

**Type** *float*

## 5.13.2 pyblp.data

Locations of example data that are included in the package for convenience.

`pyblp.data.NEVO_PRODUCTS_LOCATION`

Location of a CSV file containing the fake cereal product data from *Nevo (2000)*. The file includes the same pre-computed excluded instruments used in the original paper.

**Type** *str*

`pyblp.data.NEVO_AGENTS_LOCATION`

Location of a CSV file containing the fake cereal agent data. Included in the file are Monte Carlo weights and draws along with demographics from the original paper.

**Type** *str*

`pyblp.data.BLP_PRODUCTS_LOCATION`

Location of a CSV file containing the automobile product data extracted by *Andrews, Gentzkow, and Shapiro (2017)* from the original GAUSS code for *Berry, Levinsohn, and Pakes (1999)*, which is commonly assumed to be the same data used in *Berry, Levinsohn, and Pakes (1995)*.

The file also includes a set of excluded instruments. First, “sums of characteristics” BLP instruments from the original paper were computed with `build_blp_instruments()`. The examples section in the documentation for this function shows how to construct these instruments from scratch. As in the original paper, the “rival” instrument constructed from the `trend` variable was excluded due to collinearity issues, and the `mpd` variable was added to the set of excluded instruments for supply.

**Type** *str*

`pyblp.data.BLP_AGENTS_LOCATION`

Location of a CSV file containing the automobile agent data. Included in the file are the importance sampling weights and draws along with the income demographic from the original paper.

**Type** *str*

## Examples

---

The [online version](#) of the following section may be easier to read.

---

## Loading Data Example

```
import pyblp

pyblp.__version__

'0.10.1'
```

Any number of functions can be used to load the example data into memory. In this example, we'll first use [NumPy](#).

```
import numpy as np
blp_product_data = np.recfromcsv(pyblp.data.BLP_PRODUCTS_LOCATION, encoding='utf-8')
blp_agent_data = np.recfromcsv(pyblp.data.BLP_AGENTS_LOCATION, encoding='utf-8')
```

Record arrays can be cumbersome to manipulate. A more flexible alternative is the [pandas DataFrame](#). Unlike NumPy, pyblp does not directly depend on pandas, but it can be useful when manipulating data.

```
import pandas as pd
blp_product_data = pd.read_csv(pyblp.data.BLP_PRODUCTS_LOCATION)
blp_agent_data = pd.read_csv(pyblp.data.BLP_AGENTS_LOCATION)
```

Another benefit of DataFrame objects is that they display nicely in Jupyter notebooks.

```
blp_product_data.head()

  market_ids clustering_ids  car_ids  firm_ids region  shares  prices \
0         1971      AMGREM71     129        15     US  0.001051  4.935802
1         1971      AMHORN71     130        15     US  0.000670  5.516049
2         1971      AMJAVL71     132        15     US  0.000341  7.108642
3         1971      AMMATA71     134        15     US  0.000522  6.839506
4         1971      AMAMBS71     136        15     US  0.000442  8.928395

  hpwt  air  mpd  ...  supply_instruments2  supply_instruments3 \
0  0.528997  0  1.888146  ...                0.0                1.705933
1  0.494324  0  1.935989  ...                0.0                1.680910
2  0.467613  0  1.716799  ...                0.0                1.801067
3  0.426540  0  1.687871  ...                0.0                1.818061
```

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```

4  0.452489    0  1.504286  ...                0.0                1.933210

  supply_instruments4  supply_instruments5  supply_instruments6  \
0      1.595656                87.0                -61.959985
1      1.490295                87.0                -61.959985
2      1.357703                87.0                -61.959985
3      1.261347                87.0                -61.959985
4      1.237365                87.0                -61.959985

  supply_instruments7  supply_instruments8  supply_instruments9  \
0      0.0                46.060389                29.786989
1      0.0                46.060389                29.786989
2      0.0                46.060389                29.786989
3      0.0                46.060389                29.786989
4      0.0                46.060389                29.786989

  supply_instruments10  supply_instruments11
0      0.0                1.888146
1      0.0                1.935989
2      0.0                1.716799
3      0.0                1.687871
4      0.0                1.504286

```

[5 rows x 33 columns]

blp\_agent\_data.head()

```

  market_ids  weights  nodes0  nodes1  nodes2  nodes3  nodes4  \
0      1971  0.000543  1.192188  0.478777  0.980830 -0.824410  2.473301
1      1971  0.000723  1.497074 -2.026204 -1.741316  1.412568 -0.747468
2      1971  0.000544  1.438081  0.813280 -1.749974 -1.203509  0.049558
3      1971  0.000701  1.768655 -0.177453  0.286602  0.391517  0.683669
4      1971  0.000549  0.849970 -0.135337  0.735920  1.036247 -1.143436

  income
0  109.560369
1   45.457314
2  127.146548
3   22.604045
4  170.226032

```

*This tutorial* demonstrates how the instruments included in this dataset can be constructed from scratch.

## 5.14 Exceptions

When errors occur, they will either be displayed as warnings or raised as exceptions.

<code>exceptions.MultipleErrors</code>	Multiple errors that occurred around the same time.
<code>exceptions.NonpositiveCostsError</code>	Encountered nonpositive marginal costs in a log-linear specification.
<code>exceptions.NonpositiveSyntheticCostsError</code>	Encountered nonpositive synthetic marginal costs in a log-linear specification.
<code>exceptions.InvalidParameterCovariancesError</code>	Failed to compute standard errors because of invalid estimated covariances of GMM parameters.
<code>exceptions.InvalidMomentCovariancesError</code>	Failed to compute a weighting matrix because of invalid estimated covariances of GMM moments.
<code>exceptions.DeltaNumericalError</code>	Encountered a numerical error when computing $\delta$ .
<code>exceptions.CostsNumericalError</code>	Encountered a numerical error when computing marginal costs.
<code>exceptions.MicroMomentsNumericalError</code>	Encountered a numerical error when computing micro moments.
<code>exceptions.XiByThetaJacobianNumericalError</code>	Encountered a numerical error when computing the Jacobian of $\xi$ (equivalently, of $\delta$ ) with respect to $\theta$ .
<code>exceptions.OmegaByThetaJacobianNumericalError</code>	Encountered a numerical error when computing the Jacobian of $\omega$ (equivalently, of transformed marginal costs) with respect to $\theta$ .
<code>exceptions.MicroMomentsByThetaJacobianNumericalError</code>	Encountered a numerical error when computing the Jacobian of micro moments with respect to $\theta$ .
<code>exceptions.MicroMomentCovariancesNumericalError</code>	Encountered a numerical error when computing micro moment covariances.
<code>exceptions.SyntheticPricesNumericalError</code>	Encountered a numerical error when computing synthetic prices.
<code>exceptions.SyntheticSharesNumericalError</code>	Encountered a numerical error when computing synthetic shares.
<code>exceptions.SyntheticDeltaNumericalError</code>	Encountered a numerical error when computing the synthetic $\delta$ .
<code>exceptions.SyntheticCostsNumericalError</code>	Encountered a numerical error when computing synthetic marginal costs.
<code>exceptions.SyntheticMicroMomentsNumericalError</code>	Encountered a numerical error when computing synthetic micro moments.
<code>exceptions.EquilibriumRealizationNumericalError</code>	Encountered a numerical error when solving for a realization of equilibrium prices and shares.
<code>exceptions.PostEstimationNumericalError</code>	Encountered a numerical error when computing a post-estimation output.
<code>exceptions.AbsorptionError</code>	A fixed effect absorption procedure failed to properly absorb fixed effects.
<code>exceptions.ClippedSharesError</code>	Shares were clipped during the final iteration of the fixed point routine for computing $\delta$ .
<code>exceptions.ThetaConvergenceError</code>	The optimization routine failed to converge.
<code>exceptions.DeltaConvergenceError</code>	The fixed point computation of $\delta$ failed to converge.
<code>exceptions.SyntheticPricesConvergenceError</code>	The fixed point computation of synthetic prices failed to converge.
<code>exceptions.SyntheticDeltaConvergenceError</code>	The fixed point computation of the synthetic $\delta$ failed to converge.

Continued on next page

Table 43 – continued from previous page

<code>exceptions.EquilibriumPricesConvergenceError</code>	The fixed point computation of equilibrium prices failed to converge.
<code>exceptions.ObjectiveReversionError</code>	Reverted a problematic GMM objective value.
<code>exceptions.GradientReversionError</code>	Reverted problematic elements in the GMM objective gradient.
<code>exceptions.DeltaReversionError</code>	Reverted problematic elements in $\delta$ .
<code>exceptions.CostsReversionError</code>	Reverted problematic marginal costs.
<code>exceptions.MicroMomentsReversionError</code>	Reverted problematic micro moments.
<code>exceptions.XiByThetaJacobianReversionError</code>	Reverted problematic elements in the Jacobian of $\xi$ (equivalently, of $\delta$ ) with respect to $\theta$ .
<code>exceptions.OmegaByThetaJacobianReversionError</code>	Reverted problematic elements in the Jacobian of $\omega$ (equivalently, of transformed marginal costs) with respect to $\theta$ .
<code>exceptions.MicroMomentsByThetaJacobianReversionError</code>	Reverted problematic elements in the Jacobian of micro moments with respect to $\theta$ .
<code>exceptions.HessianEigenvaluesError</code>	Failed to compute eigenvalues for the GMM objective's (reduced) Hessian matrix.
<code>exceptions.FittedValuesInversionError</code>	Failed to invert an estimated covariance when computing fitted values.
<code>exceptions.SharesByXiJacobianInversionError</code>	Failed to invert a Jacobian of shares with respect to $\xi$ when computing the Jacobian of $\xi$ (equivalently, of $\delta$ ) with respect to $\theta$ .
<code>exceptions.IntraFirmJacobianInversionError</code>	Failed to invert an intra-firm Jacobian of shares with respect to prices when computing $\eta$ .
<code>exceptions.LinearParameterCovariancesInversionError</code>	Failed to invert an estimated covariance matrix of linear parameters.
<code>exceptions.GMMParameterCovariancesInversionError</code>	Failed to invert an estimated covariance matrix of GMM parameters.
<code>exceptions.GMMMomentCovariancesInversionError</code>	Failed to invert an estimated covariance matrix of GMM moments.

### 5.14.1 `pyblp.exceptions.MultipleErrors`

**class** `pyblp.exceptions.MultipleErrors`  
Multiple errors that occurred around the same time.

### 5.14.2 `pyblp.exceptions.NonpositiveCostsError`

**class** `pyblp.exceptions.NonpositiveCostsError`  
Encountered nonpositive marginal costs in a log-linear specification.

This problem can sometimes be mitigated by bounding costs from below, choosing more reasonable initial parameter values, setting more conservative parameter bounds, or using a linear costs specification.

### 5.14.3 `pyblp.exceptions.NonpositiveSyntheticCostsError`

**class** `pyblp.exceptions.NonpositiveSyntheticCostsError`  
Encountered nonpositive synthetic marginal costs in a log-linear specification.

This problem can sometimes be mitigated by more reasonable initial parameter values or using a linear costs specification.



#### 5.14.4 `pyblp.exceptions.InvalidParameterCovariancesError`

**class** `pyblp.exceptions.InvalidParameterCovariancesError`

Failed to compute standard errors because of invalid estimated covariances of GMM parameters.

#### 5.14.5 `pyblp.exceptions.InvalidMomentCovariancesError`

**class** `pyblp.exceptions.InvalidMomentCovariancesError`

Failed to compute a weighting matrix because of invalid estimated covariances of GMM moments.

#### 5.14.6 `pyblp.exceptions.DeltaNumericalError`

**class** `pyblp.exceptions.DeltaNumericalError`

Encountered a numerical error when computing  $\delta$ .

This problem is often due to prior problems, overflow, or nonpositive shares, and can sometimes be mitigated by choosing smaller initial parameter values, setting more conservative bounds on parameters or shares, rescaling data, removing outliers, changing the floating point precision, or using different optimization, iteration, or integration configurations.

#### 5.14.7 `pyblp.exceptions.CostsNumericalError`

**class** `pyblp.exceptions.CostsNumericalError`

Encountered a numerical error when computing marginal costs.

This problem is often due to prior problems or overflow and can sometimes be mitigated by choosing smaller initial parameter values, setting more conservative bounds, rescaling data, removing outliers, changing the floating point precision, or using different optimization or cost configurations.

#### 5.14.8 `pyblp.exceptions.MicroMomentsNumericalError`

**class** `pyblp.exceptions.MicroMomentsNumericalError`

Encountered a numerical error when computing micro moments.

This problem is often due to prior problems, overflow, or nonpositive shares, and can sometimes be mitigated by choosing smaller initial parameter values, setting more conservative bounds, rescaling data, removing outliers, changing the floating point precision, or using different optimization, iteration, or integration configurations.

#### 5.14.9 `pyblp.exceptions.XiByThetaJacobianNumericalError`

**class** `pyblp.exceptions.XiByThetaJacobianNumericalError`

Encountered a numerical error when computing the Jacobian of  $\xi$  (equivalently, of  $\delta$ ) with respect to  $\theta$ .

This problem is often due to prior problems, overflow, or nonpositive shares, and can sometimes be mitigated by choosing smaller initial parameter values, setting more conservative bounds, rescaling data, removing outliers, changing the floating point precision, or using different optimization, iteration, or integration configurations.

### 5.14.10 `pyblp.exceptions.OmegaByThetaJacobianNumericalError`

**class** `pyblp.exceptions.OmegaByThetaJacobianNumericalError`

Encountered a numerical error when computing the Jacobian of  $\omega$  (equivalently, of transformed marginal costs) with respect to  $\theta$ .

This problem is often due to prior problems or overflow, and can sometimes be mitigated by choosing smaller initial parameter values, setting more conservative bounds, rescaling data, removing outliers, changing the floating point precision, or using different optimization or cost configurations.

### 5.14.11 `pyblp.exceptions.MicroMomentsByThetaJacobianNumericalError`

**class** `pyblp.exceptions.MicroMomentsByThetaJacobianNumericalError`

Encountered a numerical error when computing the Jacobian of micro moments with respect to  $\theta$ .

### 5.14.12 `pyblp.exceptions.MicroMomentCovariancesNumericalError`

**class** `pyblp.exceptions.MicroMomentCovariancesNumericalError`

Encountered a numerical error when computing micro moment covariances.

### 5.14.13 `pyblp.exceptions.SyntheticPricesNumericalError`

**class** `pyblp.exceptions.SyntheticPricesNumericalError`

Encountered a numerical error when computing synthetic prices.

This problem is often due to prior problems or overflow and can sometimes be mitigated by making sure that the specified parameters are reasonable. For example, the parameters on prices should generally imply a downward sloping demand curve.

### 5.14.14 `pyblp.exceptions.SyntheticSharesNumericalError`

**class** `pyblp.exceptions.SyntheticSharesNumericalError`

Encountered a numerical error when computing synthetic shares.

This problem is often due to prior problems or overflow and can sometimes be mitigated by making sure that the specified parameters are reasonable. For example, the parameters on prices should generally imply a downward sloping demand curve.

### 5.14.15 `pyblp.exceptions.SyntheticDeltaNumericalError`

**class** `pyblp.exceptions.SyntheticDeltaNumericalError`

Encountered a numerical error when computing the synthetic  $\delta$ .

This problem is often due to prior problems, overflow, or nonpositive shares, and can sometimes be mitigated by making sure that the specified parameters are reasonable.

### 5.14.16 `pyblp.exceptions.SyntheticCostsNumericalError`

**class** `pyblp.exceptions.SyntheticCostsNumericalError`

Encountered a numerical error when computing synthetic marginal costs.

This problem is often due to prior problems or overflow and can sometimes be mitigated by making sure that the specified parameters are reasonable.

### 5.14.17 `pyblp.exceptions.SyntheticMicroMomentsNumericalError`

**class** `pyblp.exceptions.SyntheticMicroMomentsNumericalError`

Encountered a numerical error when computing synthetic micro moments.

### 5.14.18 `pyblp.exceptions.EquilibriumRealizationNumericalError`

**class** `pyblp.exceptions.EquilibriumRealizationNumericalError`

Encountered a numerical error when solving for a realization of equilibrium prices and shares.

### 5.14.19 `pyblp.exceptions.PostEstimationNumericalError`

**class** `pyblp.exceptions.PostEstimationNumericalError`

Encountered a numerical error when computing a post-estimation output.

### 5.14.20 `pyblp.exceptions.AbsorptionError`

**class** `pyblp.exceptions.AbsorptionError`

A fixed effect absorption procedure failed to properly absorb fixed effects.

Consider configuring absorption options or choosing a different absorption method. For information about absorption options and defaults, refer to the PyHDFE package's documentation.

### 5.14.21 `pyblp.exceptions.ClippedSharesError`

**class** `pyblp.exceptions.ClippedSharesError`

Shares were clipped during the final iteration of the fixed point routine for computing  $\delta$ .

### 5.14.22 `pyblp.exceptions.ThetaConvergenceError`

**class** `pyblp.exceptions.ThetaConvergenceError`

The optimization routine failed to converge.

This problem can sometimes be mitigated by choosing more reasonable initial parameter values, setting more conservative bounds, or configuring other optimization settings.

### 5.14.23 `pyblp.exceptions.DeltaConvergenceError`

**class** `pyblp.exceptions.DeltaConvergenceError`

The fixed point computation of  $\delta$  failed to converge.

This problem can sometimes be mitigated by increasing the maximum number of fixed point iterations, increasing the fixed point tolerance, choosing more reasonable initial parameter values, setting more conservative parameter or share bounds, or using different iteration or optimization configurations.

### 5.14.24 `pyblp.exceptions.SyntheticPricesConvergenceError`

**class** `pyblp.exceptions.SyntheticPricesConvergenceError`

The fixed point computation of synthetic prices failed to converge.

This problem can sometimes be mitigated by increasing the maximum number of fixed point iterations, increasing the fixed point tolerance, configuring other iteration settings, or making sure the specified parameters are reasonable. For example, the parameters on prices should generally imply a downward sloping demand curve.

### 5.14.25 `pyblp.exceptions.SyntheticDeltaConvergenceError`

**class** `pyblp.exceptions.SyntheticDeltaConvergenceError`

The fixed point computation of the synthetic  $\delta$  failed to converge.

This problem can sometimes be mitigated by increasing the maximum number of fixed point iterations, increasing the fixed point tolerance, choosing more reasonable parameter values, or using a different iteration configuration.

### 5.14.26 `pyblp.exceptions.EquilibriumPricesConvergenceError`

**class** `pyblp.exceptions.EquilibriumPricesConvergenceError`

The fixed point computation of equilibrium prices failed to converge.

This problem can sometimes be mitigated by increasing the maximum number of fixed point iterations, increasing the fixed point tolerance, or configuring other iteration settings.

### 5.14.27 `pyblp.exceptions.ObjectiveReversionError`

**class** `pyblp.exceptions.ObjectiveReversionError`

Reverted a problematic GMM objective value.

### 5.14.28 `pyblp.exceptions.GradientReversionError`

**class** `pyblp.exceptions.GradientReversionError`

Reverted problematic elements in the GMM objective gradient.

### 5.14.29 `pyblp.exceptions.DeltaReversionError`

**class** `pyblp.exceptions.DeltaReversionError`

Reverted problematic elements in  $\delta$ .

### 5.14.30 `pyblp.exceptions.CostsReversionError`

**class** `pyblp.exceptions.CostsReversionError`  
Reverted problematic marginal costs.

### 5.14.31 `pyblp.exceptions.MicroMomentsReversionError`

**class** `pyblp.exceptions.MicroMomentsReversionError`  
Reverted problematic micro moments.

### 5.14.32 `pyblp.exceptions.XiByThetaJacobianReversionError`

**class** `pyblp.exceptions.XiByThetaJacobianReversionError`  
Reverted problematic elements in the Jacobian of  $\xi$  (equivalently, of  $\delta$ ) with respect to  $\theta$ .

### 5.14.33 `pyblp.exceptions.OmegaByThetaJacobianReversionError`

**class** `pyblp.exceptions.OmegaByThetaJacobianReversionError`  
Reverted problematic elements in the Jacobian of  $\omega$  (equivalently, of transformed marginal costs) with respect to  $\theta$ .

### 5.14.34 `pyblp.exceptions.MicroMomentsByThetaJacobianReversionError`

**class** `pyblp.exceptions.MicroMomentsByThetaJacobianReversionError`  
Reverted problematic elements in the Jacobian of micro moments with respect to  $\theta$ .

### 5.14.35 `pyblp.exceptions.HessianEigenvaluesError`

**class** `pyblp.exceptions.HessianEigenvaluesError`  
Failed to compute eigenvalues for the GMM objective's (reduced) Hessian matrix.

### 5.14.36 `pyblp.exceptions.FittedValuesInversionError`

**class** `pyblp.exceptions.FittedValuesInversionError`  
Failed to invert an estimated covariance when computing fitted values.  
There are probably collinearity issues.

### 5.14.37 `pyblp.exceptions.SharesByXiJacobianInversionError`

**class** `pyblp.exceptions.SharesByXiJacobianInversionError`  
Failed to invert a Jacobian of shares with respect to  $\xi$  when computing the Jacobian of  $\xi$  (equivalently, of  $\delta$ ) with respect to  $\theta$ .

### 5.14.38 `pyblp.exceptions.IntraFirmJacobianInversionError`

**class** `pyblp.exceptions.IntraFirmJacobianInversionError`  
Failed to invert an intra-firm Jacobian of shares with respect to prices when computing  $\eta$ .

### 5.14.39 `pyblp.exceptions.LinearParameterCovariancesInversionError`

**class** `pyblp.exceptions.LinearParameterCovariancesInversionError`  
Failed to invert an estimated covariance matrix of linear parameters.  
One or more data matrices may be highly collinear.

### 5.14.40 `pyblp.exceptions.GMMPParameterCovariancesInversionError`

**class** `pyblp.exceptions.GMMPParameterCovariancesInversionError`  
Failed to invert an estimated covariance matrix of GMM parameters.  
One or more data matrices may be highly collinear.

### 5.14.41 `pyblp.exceptions.GMMMomentCovariancesInversionError`

**class** `pyblp.exceptions.GMMomentCovariancesInversionError`  
Failed to invert an estimated covariance matrix of GMM moments.  
One or more data matrices may be highly collinear.

## REFERENCES

This page contains a full list of references cited in the documentation, including the original work of *Berry, Levinsohn, and Pakes (1995)*. If you use PyBLP in your research, we ask that you also cite the below *Conlon and Gortmaker (2020)*, which describes the advances implemented in the package.

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## **Part II**

# **Developer Documentation**





## CONTRIBUTING

Please use the [GitHub issue tracker](#) to report bugs or to request features. Contributions are welcome. Examples include:

- Code optimizations.
- Documentation improvements.
- Alternate formulations that have been implemented in the literature but not in PyBLP.



## TESTING

Testing is done with the `tox` automation tool, which runs a `pytest`-backed test suite in the `tests/` directory. This [FAQ](#) contains some useful information about how to use `tox` on Windows.

### 9.1 Testing Requirements

In addition to the installation requirements for the package itself, running tests and building documentation requires additional packages specified by the `tests` and `docs` extras in `setup.py`, along with any other explicitly specified deps in `tox.ini`.

The full suite of tests also requires installation of the following software:

- [Artleys Knitro](#) version 10.3 or newer: testing optimization routines.
- [MATLAB](#): comparing sparse grids with those created by the function `nwspgr` created by Florian Heiss and Viktor Winschel, which must be included in a directory on the MATLAB path.

If software is not installed, its associated tests will be skipped. Additionally, some tests that require support for extended precision will be skipped if on the platform running the tests, `numpy.longdouble` has the same precision as `numpy.float64`. This tends to be the case on Windows.

### 9.2 Running Tests

Defined in `tox.ini` are environments that test the package under different python versions, check types, enforce style guidelines, verify the integrity of the documentation, and release the package. First, `tox` should be installed on top of an Anaconda installation. The following command can be run in the top-level `pyblp` directory to run all testing environments:

```
tox
```

You can choose to run only one environment, such as the one that builds the documentation, with the `-e` flag:

```
tox -e docs
```

### 9.3 Test Organization

Fixtures, which are defined in `tests.conftest`, configure the testing environment and simulate problems according to a range of specifications.

Most BLP-specific tests in `tests.test_blp` verify properties about results obtained by solving the simulated problems under various parameterizations. Examples include:

- Reasonable formulations of problems should give rise to estimated parameters that are close to their true values.
- Cosmetic changes such as the number of processes should not change estimates.
- Post-estimation outputs should satisfy certain properties.
- Optimization routines should behave as expected.
- Derivatives computed with finite differences should approach analytic derivatives.

Tests of generic utilities in `tests.test_formulation`, `tests.test_integration`, `tests.test_iteration`, and `tests.test_optimization` verify that matrix formulation, integral approximation, fixed point iteration, and nonlinear optimization all work as expected. Example include:

- Nonlinear formulas give rise to expected matrices and derivatives.
- Gauss-Hermite integrals are better approximated with quadrature based on Gauss-Hermite rules than with Monte Carlo integration.
- To solve a fixed point iteration problem for which it was developed, SQUAREM requires fewer fixed point evaluations than does simple iteration.
- All optimization routines manage to solve a well-known optimization problem under different parameterizations.

## VERSION NOTES

These notes will only include major changes.

### 10.1 0.10

- Simplify micro moment API
- Second choice or diversion micro moments
- Add share clipping to make fixed point more robust
- Report covariance matrix estimates in addition to Cholesky root
- Approximation to the pure characteristics model
- Add option to always use finite differences

### 10.2 0.9

- More control over matrices of instruments
- Split off fixed effect absorption into companion package PyHDFE
- Scrambled Halton and Modified Latin Hypercube Sampling (MLHS) integration
- Importance sampling
- Quantity dependent marginal costs
- Speed up various matrix construction routines
- Option to do initial GMM update at starting values
- Update BLP example data to better replicate original paper
- Lognormal random coefficients
- Removed outdated default parameter bounds
- Change default objective scaling for more comparable objective values across problem sizes
- Add post-estimation routines to simplify integration error comparison

## 10.3 0.8

- Micro moments that match product and agent characteristic covariances
- Extended use of pseudo-inverses
- Added more information to error messages
- More flexible simulation interface
- Alternative way to simulate data with specified prices and shares
- Tests of overidentifying and model restrictions
- Report projected gradients and reduced Hessians
- Change objective gradient scaling
- Switch to a lower-triangular covariance matrix to fix a bug with off-diagonal parameters

## 10.4 0.7

- Support more fixed point and optimization solvers
- Hessian computation with finite differences
- Simplified interface for firm changes
- Construction of differentiation instruments
- Add collinearity checks
- Update notation and explanations

## 10.5 0.6

- Optimal instrument estimation
- Structured all results as classes
- Additional information in progress reports
- Parametric bootstrapping of post-estimation outputs
- Replaced all examples in the documentation with Jupyter notebooks
- Updated the instruments for the BLP example problem
- Improved support for multiple equation GMM
- Made concentrating out linear parameters optional
- Better support for larger nesting parameters
- Improved robustness to overflow

## 10.6 0.5

- Estimation of nesting parameters
- Performance improvements for matrix algebra and matrix construction
- Support for Python 3.7
- Computation of reasonable default bounds on nonlinear parameters
- Additional information in progress updates
- Improved error handling and documentation
- Simplified multiprocessing interface
- Cancelled out delta in the nonlinear contraction to improve performance
- Additional example data and improvements to the example problems
- Cleaned up covariance estimation
- Added type annotations and overhauled the testing suite

## 10.7 0.4

- Estimation of a Logit benchmark model
- Support for fixing of all nonlinear parameters
- More efficient two-way fixed effect absorption
- Clustered standard errors

## 10.8 0.3

- Patsy- and SymPy-backed R-style formula API
- More informative errors and displays of information
- Absorption of arbitrary fixed effects
- Reduction of memory footprint

## 10.9 0.2

- Improved support for longdouble precision
- Custom ownership matrices
- New benchmarking statistics
- Supply-side gradient computation
- Improved configuration for the automobile example problem

## 10.10 0.1

- Initial release



## **Part III**

# **Indices**



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